Funding Public Goods Through Dedicated Taxes on Private Goods

Nathan W. Chan and Matthew J. Kotchen

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Abstract

This paper examines positive and normative consequences of dedicated taxes, which entail taxing a private good in order to finance the provision of a public good. Our approach differs from that in the classic public finance literature because of its setup in a game-theoretic model of private provision of an impure public good. We begin by showing that imposition of a dedicated tax either increases or decreases demand for the taxed good. We then derive intuitive conditions showing why the optimal dedicated tax can never achieve the Pareto optimal allocation that is possible under lump-sum taxation. It can, however, generate a conditionally efficient equilibrium with comparatively more or less of the public good, depending, in part, on whether the public good and the taxed private good are Hicksian complements or substitutes, respectively. We also show a neutrality result: when individuals have the opportunity to make direct donations, dedicated taxes that are sufficiently low will have no effect on the equilibrium allocation.
1 Introduction

This paper examines potential advantages and disadvantages of financing the provision of a public good by taxing a related private good. Within the theoretical literature on financing public goods, common mechanisms that rely on centralized coordination are income or wealth taxes (often lump sum) or subsidies on private provision. Other approaches rely on the benefits principle, which suggests that individuals who benefit more from the public good should pay more for its provision. Toll roads provide a common example. Another example is that visitors to National Parks pay more through admission fees.¹ There are, however, reasons why the direct benefits principle might not be desirable in many contexts, including distributional equity and administrative feasibility.

These concerns often lead to ideas about taxing related goods, a notion we refer to as a dedicated tax throughout the present paper. For example, in lieu of monitoring the distance that drivers travel on public roads for purposes of taxation, gasoline taxes are often used to finance transportation infrastructure. Dedicated taxes are also considered in the context of parks and public lands. Rather than charging high admission fees, public lands and parks can be financed to some degree through taxes on related goods, such as “gear taxes” on outdoor equipment and hunting licenses.² The intuitive appeal underlying such policies and proposals is that taxing seemingly related goods or services has advantages for financing public goods. In what follows, we provide a theoretical analysis to evaluate this intuition. In doing so, we develop an approach for examining the positive and normative consequences of using dedicated taxes to finance public goods.

Our analysis is related to the seminal literature in public finance on the optimal supply of public goods when financed through distortionary taxes (e.g., Diamond & Mirrlees, 1971; Stiglitz & Dasgupta, 1971; Atkinson & Stern, 1974). Ballard &

¹Included in this volume are papers by Ji et al. (2020) and Lupi et al. (2020) that provide empirical analyses of user fees for public lakes and beaches, respectively.
²Papers by Banzhaf & Smith (2020) and Walls (2020), which are also in this volume, provide background on the history of funding for public lands in the United States, including a discussion of excises on hunting and fishing gear and proposals for broader based gear taxes.
Fullerton (1992) summarize how the conditions can be understood by the way that the social planner’s solution differs from the foundational Samuelson (1954) condition. While the Samuelson condition can be satisfied with public good provision through lump-sum taxation, others taxes create a distortion, i.e., a marginal cost of funds, on relative prices that prevent the planner from achieving the first-best allocation. Part of our contribution is to show related results in the context of an impure public good model. By explicitly linking the consumption of a private good with provision of a public good, dedicated taxes create an impure public good equivalent to that first analyzed by Cornes and Sandler (1984; 1994; 1996). Using this framework, we are able to show in a direct and transparent way the incentives that dedicated taxes create, their efficiency consequences, and their potential scope for financing the provision of public goods.

We establish four main results. First, we show the conditions under which imposition of a dedicated tax can either increase or decrease demand for the taxed good. Second, we derive intuitive conditions showing how the Nash equilibrium under an optimal dedicated tax cannot achieve the Pareto optimal allocation that arises with the possibility for lump-sum taxation. Third, we show that the equilibrium level of the public good can exceed or fall short of the Pareto optimal level, depending on whether the public good and taxed private good are Hicksian complements or substitutes, respectively. Finally, we show a neutrality result whereby dedicated taxes that are sufficiently low will have no effect on the equilibrium allocation if individuals have the opportunity to simultaneously make voluntary contributions.

The remainder of the paper is organized as follows. The next section introduces our analytical approach of using virtual prices and income to establish comparative static results. We show how changes in the exogenously given level of a public good affects demand for private goods. Section 3 works through the Pareto efficient benchmark and its implementability with lump-sum taxes. Section 4 considers imposition of a dedicated tax and its consequences for individual behavior and the existence and uniqueness of a Nash equilibrium. Section 5 analyzes properties of the optimal dedicated tax and compares them to the Pareto efficient allocation. Section 6 generalizes
the setup to allow for the possibility of direct donations and establishes the neutrality result. Section 7 provides a concluding discussion.

2 Private and Public Goods

To establish some preliminary intuition, we begin with a basic utility maximization problem where a representative individual chooses consumption of private goods while taking the level of a public good as exogenously given. In particular, individual $i$ solves

$$\max_{x_1^i, x_2^i} U^i(x_1^i, x_2^i, X_3) \text{ s.t. } p_1 x_1^i + p_2 x_2^i = w_i^i \text{ and } X_3 = \bar{X}_3,$$

where $x_1^i$ and $x_2^i$ are private goods with the respective prices, $w_i^i$ is the individual’s wealth, and $X_3$ is a public good that is exogenously provided at level $\bar{X}_3$. We assume for the time being there is no opportunity for the individual to privately provide the public good. Because we are focusing initially on a single individual, we also drop superscripts for now. The solution to (1) can be written as demand functions $\hat{x}_j(p_1, p_2, \bar{X}_3, w)$ for good $j = 1, 2$.

Although the basic setup does not allow individuals to privately provide the public good, we can derive the individual’s marginal willingness to pay (WTP) for $X_3$. Solving the dual of (1) yields an expenditure function $e(p_1, p_2, \bar{X}_3, U^0) = w$. It follows that the individual’s marginal WTP for the public good, denoted $\pi_3$, will itself be a function of the exogenous parameters and satisfy

$$\pi_3(p_1, p_2, \bar{X}_3, w) = \frac{\partial e(p_1, p_2, \bar{X}_3, U^0)}{\partial \bar{X}_3}.$$ 

(2)

This expression indicates how marginal WTP is equal to the compensating change in income for a marginal change in the quantity of the public good.

We now consider how changes in the exogenously provided level of the public good affects demand for the private goods. That is, we are interested in what determines the sign of $\partial \hat{x}_j/\partial \bar{X}_3$. While these result are interesting in their own right, the approach that we employ for showing them helps provide the basis for the methods we employ
in subsequent sections.

We derive the comparative static results in terms of familiar price and income effects using the notations of virtual prices and income. The first step is to consider an alternative utility maximization problem where the individual can choose the aggregate level of the public good $X_3$ at a price equal to $\pi_3$, in addition to the private goods. In this case, the individual’s budget constraint can be rewritten as $p_1x_1 + p_2x_2 + \pi_3X_3 = w + \pi_3\tilde{X}_3$, where the right-hand side is the individual’s virtual full income and represents her endowment plus the value of public good spill-ins. Let $\mu = w + \pi_3\tilde{X}_3$ denote full income. By implicitly choosing $\pi_3$, we can satisfy the following condition for $i = 1, 2$:

$$\hat{x}_i(p_1, p_2, w, \tilde{X}_3) = x_i(p_1, p_2, \pi_3(\star), \mu(\star)). \quad (3)$$

This means that the solution to the “unrestricted” utility maximization problem will be identical to that for (1), and demand for the public good will be a the knife-edged solution right at the corner such that

$$\tilde{X}_3 = X_3(p_1, p_2, \pi_3(\star), \mu(\star)). \quad (4)$$

Note that the functions without the circumflex (“hat”) on the right-hand sides of (3) and (4) denote solutions for the unrestricted problem, and these are simply demand functions that have $p_1$, $p_2$, and the virtual magnitudes as arguments. It is worth clearly stating that the value of $\pi_3$ that satisfies (3) is equal to the marginal WTP defined in (2).

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3This is the approach used by Cornes and Sandler (1994; 1996) in their study of the impure public good model, but an earlier analysis that employs the same general approach for comparative static analysis among standard private and rationed goods is found in Madden (1991).
Using (3), the comparative statics of interest can therefore be written as

\[
\frac{\partial \hat{x}_j(p_1, p_2, w, \tilde{X}_3)}{\partial \tilde{X}_3} = \frac{\partial x_j(p_1, p_2, \pi_3(\cdot), \mu(\cdot))}{\partial \tilde{X}_3} = x_{j3} \frac{\partial \pi_3}{\partial \tilde{X}_3} + x_{j\mu} \frac{\partial \mu}{\partial \tilde{X}_3} = [C_{j3} - X_3 x_{j\mu}] \frac{\partial \pi_3}{\partial \tilde{X}_3} + x_{j\mu} \frac{\partial \mu}{\partial \tilde{X}_3},
\]

(5)

where the additional subscripts denote partial derivatives, the last equality comes from substituting in the Slutsky equation, and \(C_{jk}\) denotes the compensated demand response for good \(j\) with respect to a change in price \(k\). Notice that this equation expresses the results in standard price and income responses for a familiar (unrestricted) problem. The only things that remain to be solved for are changes in the virtual magnitudes \(\pi_3\) and \(\mu\) with respect to a change in \(\tilde{X}_3\). This is possible using Cramer’s rule and the identifying equations of the budget constraint and the level of the public good. We provide the details and solutions in Appendix A.1, which can be substituted into (5) to yield

\[
\frac{\partial \hat{x}_j}{\partial \tilde{X}_3} = \frac{C_{j3}(1 - \pi_3 X_3 \mu)}{C_{33}} + \pi_3 x_{j\mu}.
\]

(6)

Several results follow. If all goods are normal in the usual unrestricted sense (i.e., \(X_{3\mu} > 0\) and \(x_{i\mu} > 0\) for \(i = 1, 2\)), then the full income effects on demand are not only positive, but \(1 - \pi_3 X_{3\mu} > 0\) because a unit increase in income must be spent on all goods. This means that (6) is positive if \(x_i\) and \(X_3\) are net complements, whereas the sign is ambiguous if the two goods are net substitutes. With the special case of quasilinear preferences of the form \(x_k + F(x_j, X_3)\), equation (6) is positive or negative if \(x_j\) and the public good are net complements or substitutes, respectively.\(^4\)

\(^4\)These results provide a specific application of the finding in Madden (1991) about the symmetry of substitute and complement relationships between goods based on changes in prices or quantities of a rationed good, which in this case is the level of the public good.
3 Pareto Efficiency

We now consider an economy that consists of \( n \geq 2 \) individuals and solve for the efficient level of the public good. This provides an important point of comparison for our subsequent consideration of a dedicated tax mechanism. We assume the cost of providing the public good is unity, and without loss of generality, we normalize \( \bar{X}_3 = 0 \). Solving for the set of Pareto optimal allocations is then a matter of standard practice in public economics. All of the efficient allocations will satisfy the following first-order conditions

\[
\sum_{i=1}^{n} \frac{U_i^j}{U_i^j} = \frac{1}{p_j} \text{ for } j = 1, 2 \text{ and } \frac{U_i^1}{U_i^2} = \frac{p_1}{p_2} \text{ for } i = 1, \ldots, n, \tag{7}
\]

where for completeness we include the derivation in Appendix A.2. The first condition is the well-known Samuelson condition, where the sum of the marginal rates of substitution between the public good and all private goods equals the corresponding price ratios. The second is the standard condition for private goods, where the marginal rates of substitution between goods equals the price ratio for all individuals.

We simplify things further by assuming identical preferences across all individuals and focusing on the symmetric allocation. The symmetry is helpful for our comparisons below and is also the allocation that a social planner would choose with equal welfare weights across individuals. With these assumptions, the unique solution will satisfy

\[
\frac{nU_3}{U_j} = \frac{1}{p_j} \text{ for } j = 1, 2 \text{ and } \frac{U_1}{U_2} = \frac{p_1}{p_2}, \tag{8}
\]

which is a special case of the conditions in (7). To be clear, this implies the same allocation of private goods for each individual \((x_1^*, x_2^*)\) and a unique level of the public good

\[
X_3^* = \sum_{i=1}^{n} [w_i - n(p_1x_1^* + p_2x_2^*)]. \tag{9}
\]

We can verify the standard result that lump-sum taxes can be used to implement the Pareto optimal allocation. With individualized taxes \( \tau_i \), each individual’s utility
maximization problem is
\[
\max_{x_1, x_2} U(x_1, x_2, X_3) \text{ s.t. } p_1 x_1 + p_2 x_2 = w_i - \tau_i \text{ and } X_3 = \sum_{i=1}^{n} \tau_i,
\]
with the corresponding first-order condition
\[
\frac{U_1(x_1, x_2, \sum_{i=1}^{n} \tau_i)}{U_2(x_1, x_2, \sum_{i=1}^{n} \tau_i)} = \frac{p_1}{p_2}.
\]
Setting each individual's tax such that \( \tau_i^* = w_i - p_1 x_1^* + p_2 x_2^* \) has two implications. The first is that \( \sum_{i=1}^{n} \tau_i^* = X_3^* \) by (9), so the public good is fully funded. The second is that \( (x_1^*, x_2^*) \) satisfies each individual's budget constraint and (10), which is equivalent to the second condition in (8). This means that imposing \( \tau_i^* \) for all \( i \) implements the Pareto efficient and symmetric allocation as an equilibrium outcome. Note the possibility for \( \tau_i^* \) to in fact be a subsidy in some cases if an individual's endowment is sufficiently low. To simplify things even further in what follows, we make the additional assumption of identical endowments \( w \) across individuals. In this case, the optimal lump-sum tax is a uniform "head" or "poll" tax that satisfies
\[
\tau^* = w - p_1 x_1^* + p_2 x_2^* = X_3^*/n.
\]

4 A Dedicated Tax

We now turn to a dedicated tax mechanism to finance the public good, which we assume applies to good \( x_2 \) without loss of generality. In particular, we model a tax rate of \( t_2 \) per unit \( x_2 \), and all proceeds are used to finance the provision of \( X_3 \).\(^5\) We first characterize individual incentives before turning to the Nash equilibrium.

\(^5\)We continue to assume that individuals are not able to make direct donations to provide the public good, though we eventually relax this assumption later in the paper.
4.1 Individual Behavior

Each individual solves the following utility maximization problem, taking the level of the public good provided by all others as given:

$$\max_{x_1, x_2} U(x_1, x_2, t_2 x_2 + \bar{X}_3) \text{ s.t. } w = p_1 x_1 + (p_2 + t_2) x_2.$$  (11)

Assuming an interior solution, the first-order conditions can be combined to

$$\frac{U_2 + t_2 U_3}{U_1} = \frac{p_2 + t_2}{p_1}. \quad (12)$$

This has an intuitive interpretation: the ratio of the marginal utilities (benefits) of the two goods equals the price ratio. The difference here from a more typical setup is that $x_2$ is linked to $X_3$ through $t_2$, which defines the relative quantities and the tax-inclusive price. Suppressing notation for prices and $w$, we can fully characterize the solution to (11) as $\hat{x}_2(t_2, \bar{X}_3)$.

An interesting feature of the setup in (11) is that for any given level of $t_2$, it is a special case of Cornes and Sandler’s (1984; 1994; 1996) impure public good model. This follows because consumption of the taxed private good becomes associated with joint production of the public good. Nevertheless, the comparative static properties of the model will differ because here the price of the jointly produced good and the technology of joint production are not independent parameters, as they are both functions of $t_2$.

It is both interesting and useful to consider how demand for $x_2$ changes with imposition of the dedicated tax. Keep in mind this is not simply a price effect, as an increase in the tax also provides the public good. We again employ virtual prices and income to derive results in terms of familiar price and income effects. In Appendix A.3, we derive the following general result:

$$\frac{\partial \hat{x}_2}{\partial t_2} = \frac{x_2(t_2 C_{22} - C_{23}) + (1 - \pi_3) \Psi + (1 - \pi_3) x_2 [(C_{33} - t_2 C_{32}) x_2 \mu + (t_2 C_{22} - C_{23}) X_{3\mu}]}{\Omega}.$$  (13)
where

\[ \Psi = C_{23}C_{32} - C_{22}C_{33} < 0 \text{ and } \Omega = t_2(C_{23} + C_{32}) - t_2^2C_{22} - C_{33} > 0, \]

and the signs of these expressions follow by negative semi-definiteness of the Slutsky matrix.\(^6\) Because (13) is rather cumbersome, we again consider the special case of quasilinear preferences to illustrate the different possibilities. Here preferences take the form \(x_1 + F(x_2, X_3)\), and we simplify even further to consider a scenario of first imposing the tax from a starting point of \(t_2 = 0\). With these simplifications, equation (13) becomes

\[ \frac{\partial \hat{x}_2}{\partial t_2} = \frac{-x_2C_{23} + (1 - \pi_3)\Psi}{-C_{33}}. \] (14)

This establishes several results about how demand for \(x_2\) will respond to imposition of a dedicated tax \(t_2\). Table 1 summarizes the qualitative results, showing how they depend on the individual’s marginal WTP for the public good at the initial allocation.\(^7\) Consider the knife-edge case where the individual’s marginal WTP exactly equals the per unit cost of providing the public good (i.e., \(\pi_3 = 1\)). If the tax-linked goods are net complements or substitutes, then demand for the private good will increase or decrease, respectively. Notice that an increase in demand for the private good is rather counter-intuitive, because imposition of a tax increases demand. Interestingly, this suggests that sellers of the good might actually benefit from imposition of the tax, which is a possibility explored in greater detail by Banzhaf & Smith (2020).

The same qualitative results occur if the marginal WTP, that is \(\pi_3\), is greater (less) than unity, and the linked goods are complements (substitutes). Ambiguity occurs when there are effects that push in different directions, that is, when the marginal WTP is less (greater) than unity, and the linked goods are complements (substitutes). Recall, however, that we are considering the special case of quasilinear preferences here to illustrate the range of possibilities; in a more general setting, these different

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\(^6\)The Slutsky matrix is equivalent to the Hessian of the expenditure function and the matrix of derivatives of the compensated demand functions, and we assume the weak inequalities implied by negative semi-definiteness hold strictly.

\(^7\)See Banzhaf & Smith (2020) for a similar set of results, though motivated with a different setup.
Table 1: The change in demand for $x_2$ given imposition of a dedicated tax $t_2$ with quasilinear preferences

<table>
<thead>
<tr>
<th>Relationship between $x_2$ and $X_3$</th>
<th>$\pi_3 &lt; 1$</th>
<th>$\pi_3 = 1$</th>
<th>$\pi_3 &gt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net complements</td>
<td>?</td>
<td>+</td>
<td>+</td>
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<tr>
<td>Net substitutes</td>
<td>-</td>
<td>-</td>
<td>?</td>
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</tbody>
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possibilities might occur in each of the cases.

4.2 Nash Equilibrium

The model’s setup implies that individuals are playing a game for any given level of the dedicated tax. We therefore consider equilibrium existence and uniqueness for any given tax rate.\(^8\) We can write each individual’s demand for private provision as $\hat{x}_3(t_2, \bar{X}_3) = t_2\hat{x}_2(t_2, \bar{X}_3)$, which is his or her best-response function.\(^9\) A Nash equilibrium is a fixed point at the intersection of all $n$ best response functions. Equivalently, a Nash equilibrium is a set of choices $x_2^i$ for all $i$ that satisfy the first order condition (12) for all $n$ individuals with $X_3 = t_2 \sum_{i=1}^{n} x_2^i$.

Kotchen (2007a) establishes a sufficient condition for equilibrium existence and uniqueness in the general impure public good model, and as noted above, the setup here is a special case for a given level of $t_2$. The condition is based on the slope of each individual’s demand for the aggregate level of the public good with respect to the provision of others. In particular, using the notation employed here, the sufficient condition is:

$$0 < \frac{\partial \hat{X}_3(t_2, \bar{X}_3)}{\partial X_3} = t_2 \frac{\partial \hat{x}_2(t_2, \bar{X}_3)}{\partial \bar{X}_3} + 1 \leq 1.$$  

(15)

This says that an increase in spill-ins must increase demand for the public good and not decrease demand for the untaxed private good $x_1$. This is essentially a

\(^8\)Our examination of the public good provision equilibrium is one way in which our analysis differs from that in Banzhaf & Smith (2020). They do, however, consider an equilibrium among suppliers, whereas we simplify the supply side of the market by assuming fixed and exogenous prices.

\(^9\)Note that each individual’s demand for the aggregate level of the public then follows by definition: $\hat{X}_3 = t_2\hat{x}_2(t_2, \bar{X}_3) + \bar{X}_3$. 
normality assumption with respect to full, virtual income. A further implication is that best-response functions have slopes less than zero and greater than $-1$, and this monotonicity combined with continuity ensures the existence of a unique fixed point.

With identical individuals, a further implication is that the equilibrium will be symmetric, and we denote it simply as $x_2^N(t_2)$ for each individual. It follows that the aggregate, equilibrium level of public good provision will satisfy

$$X_3^N(t_2) = nx_3^N(t_2) = nt_2x_2^N(t_2),$$

which shows how the solution can written in terms of each individual’s level of private provision or demand for the taxed private good.

Before turning to the optimal dedicated tax in the next section, it is helpful to establish an intermediate result here. We showed in the previous subsection that demand for $x_2$ can be increasing or decreasing in response to imposition of a dedicated tax $t_2$. The question that we consider now is whether $\partial \hat{x}_2 / \partial t_2$ in (13) can take either sign while satisfying the assumption in (15). The reason is that maintaining both possibilities creates some interesting results that we derive in the next section. In general, the answer is yes, which we show in Appendix A.4. This means that even imposing the constraint on individual behavior that is sufficient for a unique Nash equilibrium, it is still possible for $\partial \hat{x}_2 / \partial t_2 \lesssim 0$.\textsuperscript{10}

## 5 The Optimal Dedicated Tax

We now consider the optimal dedicated tax that a social planner would choose. We also compare the resulting allocation with the Pareto optimal allocation defined in

\textsuperscript{10}The possibility for $\partial \hat{x}_2 / \partial t_2 < 0$ is perhaps somewhat less surprising because as described earlier $t_2$ operates, in part, like an increase in price. But we have also discussed how it has the additional effect of increasing the level of the public good for a given level of $x_2$, and the sign of the comparative static depends on the signs and relative magnitudes of multiple price and income effects. Given the condition in (15), we show in Appendix A.5 that if $\pi \leq 1$, then assuming the two goods are normal and net substitutes is sufficient for $\partial \hat{x}_2 / \partial t_2 < 0$. However, in order to obtain the opposite sign with both goods still being normal, a necessary condition is for net complements, where in particular $0 > C_{22} > C_{32}(= C_{23}) > C_{33}$. Intuitively, this corresponds to the case where $x_2$ has a relatively small own-price response.
Section 3. It is important to recognize with any dedicated tax, the individuals continue to play a non-cooperative Nash game that the planner must take into account when choosing $t_2$.

We can write the planner’s objective as:

$$\max_{t_2} nU \left( \frac{w - (p_2 + t_2)x_2^N(t_2)}{p_1}, x_2^N(t_2), nt_2x_2^N(t_2) \right).$$

The first-order condition that defines the solution is

$$nU_3x_2^N - U_1 \frac{x_2^N}{p_1} + \left( U_2 - U_1 \frac{p_2 + t_2}{p_1} + nt_2U_3 \right) \frac{\partial x_2^N}{\partial t_2} = 0.$$

Using (12), which must also hold in equilibrium, and rearranging terms, we can rewrite this equation more compactly as

$$\frac{U_3}{U_1} \left( n + (n - 1)\varepsilon_{x_2^N, t_2} \right) = \frac{1}{p_1},$$

where $\varepsilon_{x_2^N, t_2} = \frac{t_2}{x_2^N} \frac{\partial x_2^N}{\partial t_2}$ is the elasticity representing the percentage change in each individual's equilibrium level of demand for $x_2$ given a percentage change in the tax rate. Notice that the elasticity here differs from a standard demand elasticity for two reasons. First, as mentioned previously, a change in the tax rate is not simply a change in the the price because the revenue is used to provide the public good that directly enters the individual utility functions. Second, the elasticity is for an equilibrium response and not simply a demand response, meaning that the change in behavior of all others is taken into account.\textsuperscript{11}

5.1 Efficiency

We now make comparisons between the Pareto optimal allocation and that of the optimal dedicated tax. This requires a comparison between the conditions specified

\textsuperscript{11}In Appendix A.5, we show the exact relationship between the equilibrium elasticity $\varepsilon_{x_2^N, t_2}$ and the demand response elasticity, denoted $\varepsilon_{x_2, t_2}$. In general, we find nothing that rules out the possibility for the equilibrium elasticity to take either sign, and in most cases, it will have the same sign as the demand response elasticity.
in (8) and those specified in (16) and (12). That is, for the optimal dedicated tax, we must satisfy the planner’s solution and the first-order condition for each individual because the allocation is itself an equilibrium outcome. Let us begin with the special, though somewhat trivial, case of \( n = 1 \). Using (16) and substituting it into (12), we find after rearranging that the two sets of conditions are equivalent. This means that the efficient dedicated tax can implement the Pareto optimal allocation when \( n = 1 \). The simple intuition is that the planner can set the tax to exactly balance the individual’s preferred consumption of the public good and taxed private good.

Another helpful special case to consider, albeit knife-edged, is one where \( \varepsilon_{x^N_2,t_2} = 0 \), which as shown in Appendix A.5 is equivalent to assuming the demand response \( \varepsilon_{x_2,t_2} = 0 \). Equation (16) matches the first in (8), but (12) does not match the second. Substituting (16) into (12) and rearranging yields

\[
\frac{U_2}{U_1} = \frac{p_2 + \frac{t_2(n-1)}{n}}{p_1}. \tag{17}
\]

This equation shows how even if \( \varepsilon_{x^N_2,t_2} = 0 \), the tax affects how the individual makes trade-offs between the private goods. Compared to the first-best allocation, this is a price distortion, and it arises because of the different incentives between individuals and the planner. With \( n = 1 \), there is no such distortion because all of the tax revenue is used to satisfy the individual’s own demand for the public good. With \( n \geq 2 \), however, the planner is setting the tax to account for the marginal benefit across all individuals, and the share of the tax that each individual faces for the benefit of others (and not oneself) is increasing in \( n \). Equation (17) makes clear how the additional price on \( x_2 \) is \( \frac{1}{2}t_2 \) for \( n = 2 \), then \( \frac{2}{3}t_2 \) for \( n = 3 \), and ultimately the full amount \( t_2 \) as \( n \to \infty \). In the latter case, the intuition is that the individual only has an infinitesimally small effect on the level of the public good but is taxed the full amount \( t_2 \) on her own \( x_2 \) consumption. More generally, as long as \( \varepsilon_{x^N_2,t_2} \neq 0 \), the dedicated tax will distort the trade-off between all goods, including the public good.

The preceding discussion is sufficient to establish a clear result: for \( n \geq 2 \), the optimal dedicated tax cannot achieve the Pareto optimal allocation that could arise
with lump-sum taxation. It is therefore a second-best policy. While it does provide
a mechanism for providing the public good, the linear constraint between the taxed
private good and the level of public good provision will, in general, distort the trade-
off(s) among goods.

5.2 Level of Public Good Provision

We now turn our focus to implications for the level of public good provision. This
may in fact be the primary motive for considering a dedicated tax (in contrast to
an efficiency objective), and the results provide further intuition for why a dedicated
tax does not implement the Pareto optimal allocation. Ballard & Fullerton (1992)
broadly describe an augmented Samuelson condition that applies in the context of a
distortionary tax as

\[
\sum_{i=1}^{n} MRS_i = MRT \times MCF,
\]

(18)

where the \( MCF \) is the marginal costs of funds defined in terms of the reference
private good. The \( MCF \) represents the cost to individuals per unit of revenue raised
to provide the public good. If the \( MCF = 1 \), which is the case with lump-sum
taxes, the expression reverts back to the basic Samuelson condition. Nevertheless,
whether the \( MCF \) is less than or greater than \( 1 \) has implications for the efficient
level of public good provision, conditional on the tax mechanism under consideration.
Ballard & Fullerton (1992) discuss how the \( MCF > 1 \) is typically assumed, yet
emphasize the possibility for circumstances where \( MCF < 1 \).

Both possibilities emerge clearly in the model considered here. Rearranging (16),
we can solve immediately for

\[
MCF = 1 - p_1 \frac{U_3}{U_1} (n - 1) \varepsilon_{x_2^N,t_2}.
\]

Beyond the direct cost of raising a unit of revenue (unity), there is an additional effect
based on how the equilibrium demand for \( x_2 \) responds. If demand decreases (i.e.,
the elasticity is negative), the \( MCF \) is greater than 1, where the demand response is
weighted by the utility loss to \(n-1\) individuals of less public good, which is converted to revenue via the price of the private good. If, however, demand increases (i.e., the elasticity is positive), then the MCF will be less than 1. Hence the sign of the equilibrium demand elasticity will determine whether the MCF is greater than or less than 1.\(^{12}\)

What, then, does the sign of \(\varepsilon_{x_N,t_2}\), operating through the magnitude of the MCF, imply about the equilibrium level of public good provision? In Appendix A.6, we derive the results summarized in Table 2 under the assumption of strict concavity of the utility function in all three arguments. The table compares the relative magnitudes of the equilibrium quantities for the optimal dedicated tax with those of the Pareto optimal allocation. The signs in each cell indicate whether the dedicated tax equilibrium has a higher (+) or lower (−) quantity. The different columns correspond with different ranges of \(\varepsilon_{x_N,t_2}\) and the associated magnitude of the MCF. Intuitively, we find that if the MCF ≤ 1 (or equivalently \(\varepsilon_{x_N,t_2} \geq 0\)), then the dedicated tax equilibrium level of the public good is greater than the Pareto optimal allocation. In contrast, if the MCF > 1 and \(\varepsilon_{x_N,t_2} \leq -1\), then the equilibrium level of the public good is lower. Finally, there is an intermediate case where the relative magnitudes of \(X_3\) and \(x_2\) are indeterminate, and this occurs when MCF > 1 and \(\varepsilon_{x_N,t_2} \in (-1, 0)\), that is, the equilibrium demand response is negative but inelastic.

\(^{12}\)Note that satisfying (16) means that for an interior solution, it must hold that the elasticity satisfies \(\varepsilon_{x_N,t_2} > -n/(n-1)\).
5.3 Summary

We have illustrated two main results in this section. The first is that an optimally chosen dedicated tax will not, in general, implement the Pareto optimal allocation, and the associated $MCF$ is an intuitive way to understand the result. The second is that, despite its inefficiency compared to a lump-sum tax, the optimally chosen dedicated tax can implement an equilibrium level of the public good that exceeds or falls short of the first-best level, depending on the sign and magnitude of the equilibrium tax elasticity of demand for the taxed private good. If, for example, imposing (or increasing) the dedicated tax stimulates demand for the taxed good, the $MCF$ is lower, and this creates less distortion from providing greater levels of the public good, thereby enabling more from an efficiency perspective.

6 Generalization with Direct Donations

We have thus far assumed the only way that individuals can provide the public good is through consumption of the taxed private good. In this section, we generalize the model's setup to allow the possibility for individuals to make a direct contribution to the public good, in addition to the possibility of provision through the taxed private good. The expanded choice set, with multiple channels for public good provision, is similar to that in Kotchen (2006, 2007b) and Chan & Kotchen (2014).

Prior to implementation of a dedicated tax, we can write the individual's problem as

$$\max_{x_1, x_2, x_3} U(x_1, x_2, x_3 + \bar{X}_3) \text{ s.t. } w = p_1 x_1 + p_2 x_2 + x_3,$$

where the difference here is that the individual can choose $x_3$ directly. Substituting the budget constraint into the maximand and choosing $x_2$ and $x_3$, the Kuhn-Tucker first-order conditions are

$$x_2 \geq 0, \quad -\frac{p_2}{p_1} U_1 + U_2 \leq 0, \quad \text{c.s.}$$
$$x_3 \geq 0, \quad -\frac{1}{p_1} U_1 + U_3 \leq 0, \quad \text{c.s.}$$

(19)
where c.s. denotes the complementary slackness condition. Note that the Pareto optimal allocation with this setup remains the same as defined in Section 3.

An equivalent and useful way to write the individual’s problem is with the implicit choice of the aggregate level of public good provision:

$$\max_{x_1, x_2, X_3} U(x_1, x_2, X_3) \text{ s.t. } w + \bar{X}_3 = p_1 x_1 + p_2 x_2 + X_3 \text{ and } X_3 \geq \bar{X}_3,$$

where the corresponding first-order conditions are identical to those above. Following convention for the pure public model (see Bergstrom et al. 1986), we can write the solution as

$$X_3 = f(w + \bar{X}_3),$$

where we have suppressed notation for prices. The argument in $f(\cdot)$ is the individual’s full income: wealth plus the value of public good spill-ins. The standard assumption is to assume normality of all goods, in which case $0 < f' \leq 1$, where the inequality holds strictly for an interior solution. This guarantees equilibrium existence and uniqueness. This follows because best-response functions $x_3 = f(w + \bar{X}_3) - \bar{X}_3$ are continuous and non-increasing, which gives a unique fixed point for each individual’s direct contribution. The first-order condition establishing the trade-off between $x_1$ and $x_2$, along with the budget constraint, then defines each individual’s choice of the private goods. The equilibrium level of the public good will therefore satisfy

$$\bar{X}_3 = f(w + (n-1)\bar{X}_3),$$

where each individual’s equilibrium allocation is denoted $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ and $\bar{x}_3 = \bar{X}_3/n$.

We now introduce the dedicated tax to this more general setup. The individual’s problem is

$$\max_{x_1, x_2, x_3} U(x_1, x_2, t_2 x_2 + x_3 + \bar{X}_3) \text{ s.t. } w = p_1 x_1 + (p_2 + t_2) x_2 + x_3,$$

which shows the two ways to potentially provide the public good: through consump-
tion of $x_2$, through a direct donation $x_3$, or both. In this case, the Kuhn-Tucker first order conditions are

$$x_2 \geq 0, \quad -\frac{(p_2 + t_2)}{p_1} U_1 + U_2 + t_2 U_3 \leq 0, \quad c.s.$$

$$x_3 \geq 0, \quad -\frac{1}{p_1} U_1 + U_3 \leq 0, \quad c.s.$$  \hspace{1cm} (20)

Notice that only the first condition in (20) differs from that in (19). This follows because with the opportunity to make a direct donation, the dedicated tax does not distort the price ratio between the public good and the untaxed private good.

Assuming there is an interior solution, we can substitute the second condition in (20) into the first and find that the conditions defining an equilibrium are identical to those in (19). This establishes a neutrality result: implementing a dedicated tax has no effect on the equilibrium if individuals continue to purchase the private goods and make a direct donation after the tax is imposed. The intuition is that provision through the dedicated tax crowds out direct donations one-for-one.

To see the crowding out of one’s own direct donations more directly, consider how the equilibrium level of the public good at an interior solution will satisfy

$$\bar{X}_3 = n(t_2 \bar{x}_2 + \bar{x}_3).$$

Totally differentiating, and recognizing that $d\bar{X}_3 = 0$ because of the same conditions in (19) and (20), we have

$$\frac{d\bar{x}_3}{dt_2} = -\bar{x}_2(1 + \varepsilon_{\bar{x}_2,t_2}).$$

Yet because the equilibrium allocation of all three goods does not change, it must also hold that $\varepsilon_{\bar{x}_2,t_2} = 0$, meaning that $\frac{d\bar{x}_3}{dt_2} = -\bar{x}_2$. This illustrates how direct donations are decreasing in the dedicated tax rate such that the crowding out is one-for-one. This follows because, given a change in $t_2$, the change in an individual’s contribution to the public good through consumption of the taxed private good is exactly $-\bar{x}_2$.\textsuperscript{13}

\textsuperscript{13}A well-known result in the charitable giving literature is that one-for-one crowding out no longer occurs in the model of impure altruism (Andreoni 1989, 1990). In the setting considered here, however, the crowding effects continue to hold exactly if warm-glow benefits are derived equally from the public good provision that occurs through either direct donations or the dedicated tax.
A further implication is that $t'_2 = \bar{X}_3/n\bar{x}_2$ is the level of the tax that exactly reduces direct donations to zero and maintains the same equilibrium allocation. We have thus shown complete neutrality for all $t_2 \leq t'_2$. Things change, however, for dedicated tax levels greater than $t'_2$, where an additional increase in the tax rate will render direct donations irrelevant as they are completely crowded out by tax revenues. Thus, for $t_2 > t'_2$, the model with direct donations reduces to the simpler setup in previous sections.

7 Conclusion

This paper set out to examine some of the advantages and disadvantages of using dedicated taxes to finance the provision of public goods. We show how the impure public good model provides a useful way for understanding the positive and normative consequences of dedicated taxes. We began by showing how imposition of a dedicated tax can, somewhat counter-intuitively, increase or decrease demand for the taxed good. We then showed how the optimal dedicated tax can never achieve the Pareto optimal allocation that is possible under lump-sum taxation. It can, however, generate a conditionally efficient equilibrium with comparatively more or less of the public good, depending, in part, on whether the public good and the taxed private good are Hicksian complements or substitutes, respectively. We also show a neutrality result: when individuals have the opportunity to make direct donations, dedicated taxes that are sufficiently low will have no effect on the equilibrium allocation.

Several of these results may help to illuminate the potential political economy and policy implications of dedicated taxes. For example, the possibility that dedicated taxes can stimulate demand for the taxed good suggests suppliers may actually support dedicated taxes (see also Banzhaf & Smith 2020). Indeed, imposing dedicated taxes may provide a mechanism with benefits akin to corporate social responsibility.

This follows because both alternatives are perfect substitutes with respect to public good provision at interior solutions. Nevertheless, the perfect substitutability will no longer hold if only one form of giving (e.g., direct donations) confers warm glow benefits while the other (e.g., indirect donations via tax payments) does not. In such cases, the crowding effect will be mitigated in ways consistent with that underlying the standard results in the literature.
that links public and private goods (Besley & Ghatak 2007), where all producers must meet the same standards rather than having it be met voluntarily. A similar mechanism, albeit voluntary in membership, is the 1% For the Planet initiative, where a group of business, including companies like Patagonia, donate 1% of gross sales to environmental non-profit organizations. In this case, the dedicated tax is effectively 1% of sales.

The neutrality result also highlights one of the potential risks of dedicated taxes: the crowding out of direct donations. Consider, for example, how federal funding for wildlife and habitat conservation is often disbursed contingent upon matching funds from states, as is the case with the Pittman-Robertson and Dingell-Johnson Acts. While states typically raise matching funds from hunting and fishing licenses, they also rely on contributions from private organizations, with Ducks Unlimited being a frequent and significant partner with state agencies for securing federal money. Our findings point to the potential unintended crowding out that may accompany a state’s greater reliance on dedicated taxes. If contributions from organization such as Ducks Unlimited are crowded out, then the dedicated taxes may have limited or no effect on total conservation funding. Along these lines, Walls (2020) also warns of a second dimension of crowding out, whereby dedicated taxes may crowd out allocations from general revenues.
A Appendix

A.1 Derivation of Equation (6)

We use two equations to solve for changes in the virtual magnitudes: the definition of virtual full income

\[ \mu(p_1, p_2, w, \tilde{X}_3) = w + \pi_3(p_1, p_2, w, \tilde{X}_3) \tilde{X}_3 \]

and the constraint on the level of the public good

\[ X_3(p_1, p_2, \pi_3(\cdot), \mu(\cdot)) = \tilde{X}_3. \]

Differentiating both of these with respect to \( \tilde{X}_3 \) yields

\[ \frac{\partial \mu}{\partial \tilde{X}_3} = \frac{\partial \pi_3}{\partial \tilde{X}_3} \tilde{X}_3 + \pi_3, \]

and

\[ X_{33} \frac{\partial \pi_3}{\partial \tilde{X}_3} + X_{3\mu} \frac{\partial \mu}{\partial \tilde{X}_3} = 1. \]

With these two equations, we can substitute in the Slutsky decomposition and solve for changes in the virtual magnitudes:

\[ \frac{\partial \mu}{\partial \tilde{X}_3} = \frac{1 - \pi_3 X_{3\mu}}{C_{33}} \]

and

\[ \frac{\partial \pi_3}{\partial \tilde{X}_3} = \left[ \frac{1 - \pi_3 X_{3\mu}}{C_{33}} \right] \tilde{X}_3 + \pi_3. \]

Substituting these expressions into (5) and rearranging immediately yields (6).
A.2 Derivation of Conditions (7) and (8)

Allowing heterogeneous preferences, endowments, and arbitrary welfare weights \( a_i \) for all \( i \), the set of Pareto optimal allocations must solve

\[
\max_{\{x_1^i, x_2^i\}} \sum_{i=1}^{n} a_i U^i(x_1^i, x_2^i, Y) \text{ s.t. } Y = \sum_{i=1}^{n} (w^i - p_1 x_1^i - p_2 x_2^i).
\]

The first-order conditions are

\[
\sum_{i=1}^{n} a_i p_1 U_3^i = a_i U_1^i \text{ for all } i
\]

\[
\sum_{i=1}^{n} a_i p_2 U_3^i = a_i U_2^i \text{ for all } i.
\]

The first implies that \( a_i U_1^i \) is equal for all \( i \), and the second implies that \( a_i U_2^i \) is equal for all \( i \). With a bit of rearranging, these first-order conditions immediately imply the conditions in (7). Assuming further that all individuals have identical preferences and that \( a_i = a \) for all \( i \), the first-order conditions above further imply the conditions in (8), which define the symmetric, Pareto optimal allocation.

A.3 Derivation of Equation (13)

The solution to (11) will satisfy the unrestricted solution based on virtual magnitudes such that

\[
\tilde{x}_2(t_2, \tilde{X}_3) = x_2(\pi_2(\cdot), \pi_3(\cdot), \mu(\cdot)),
\]

where the virtual magnitudes are themselves functions of the exogenous parameters \((p_1, p_2, w, \tilde{X}_3, t_2)\). The comparative static of interest can be expressed generally as

\[
\frac{\partial \tilde{x}_2}{\partial t_2} = x_{22} \frac{\partial \pi_2}{\partial t_2} + x_{23} \frac{\partial \pi_3}{\partial t_2} + x_{2\mu} \frac{\partial \mu}{\partial t_2}.
\]

(21)
The three equations that the virtual magnitudes must satisfy are based on the tax-inclusive price of $x_2$ equaling the value of what it buys in terms of virtual magnitudes

\[ p_2 + t_2 = \pi_2(\cdot) + t_2\pi_3(\cdot), \]

the quantity relationship between $x_2$ and the public good

\[ t_2x_2(\pi_2(\cdot), \pi_3(\cdot), \mu(\cdot)) = X_3(\pi_2(\cdot), \pi_3(\cdot), \mu(\cdot)) - \tilde{X}_3, \]

and the definition of virtual full income

\[ \mu(\cdot) = w + \pi_3(\cdot)\tilde{X}_3. \]

Differentiating these equations with respect to $t_2$, using Cramer’s rule and the Slutsky decomposition, we can solve for changes in each of the virtual magnitudes:

\[
\frac{\partial \pi_2}{\partial t_2} = \frac{t_2x_2 - (1 - \pi_3)(C_{33} - t_2C_{23}) - (1 - \pi_3)(t_2x_2)(t_2x_{2\mu} - X_{3\mu})}{\Omega},
\]

\[
\frac{\partial \pi_3}{\partial t_2} = \frac{-x_2 + (1 - \pi_3)(C_{32} - t_2C_{22}) + (1 - \pi_3)x_2(t_2x_{2\mu} - X_{3\mu})}{\Omega},
\]

\[
\frac{\partial \mu}{\partial t_2} = \frac{\partial \pi_3}{\partial t_2} \tilde{X}_3.
\]

Substituting these three expressions into (21), using the Slutsky decomposition again, and rearranging immediately yields equation (13).

**A.4 Verification that $\frac{\partial \tilde{x}_2}{\partial t_2} \leq 0$ Given Assumption (15)**

We have already established that

\[
\frac{\partial \tilde{x}_2}{\partial t_2} = \frac{x_2(t_2C_{22} - C_{23}) + (1 - \pi_3)\Psi + (1 - \pi_3)x_2[(C_{33} - t_2C_{32})x_{2\mu} + (t_2C_{22} - C_{23})X_{3\mu}]}{\Omega}. \]
Following the same steps for deriving a comparative static result using virtual magnitudes, it can be shown that

\[
\frac{\partial \hat{X}_3}{\partial X_3} = \frac{(t_2C_{32} - C_{33}) + t_2\pi_3 [(C_{23} - t_2C_{22})X_{3\mu} + (t_2C_{32} - C_{33})x_{2\mu}]}{\Omega},
\]

and assumption (15) requires that \(0 < \partial \hat{X}_3 / \partial \tilde{X}_3 \leq 1\). To simplify notation, and without loss of generality, we normalize \(t_2 = 1\) and let \(A \equiv C_{32} - C_{33}\) and \(B \equiv C_{23} - C_{22}\). Note that \(A + B = \Omega > 0\), and recall that \(\Psi < 0\). The two equations above simplify to

\[
\frac{\partial \hat{x}_2}{\partial t_2} = -x_2B + (1 - \pi_3)\Psi + (1 - \pi_3)x_2 [-Ax_{2\mu} - BX_{3\mu}] \quad (22)
\]

and

\[
\frac{\partial \hat{X}_3}{\partial \tilde{X}_3} = \frac{A + \pi_3 [Ax_{2\mu} + BX_{3\mu}]}{A + B}. \quad (23)
\]

Using the notation in (23), satisfying the conditions in (15) requires both of the following:

\[
A + \pi_3 [Ax_{2\mu} + BX_{3\mu}] > 0 \quad (24)
\]

and

\[
\pi_3 [Ax_{2\mu} + BX_{3\mu}] - B \leq 0. \quad (25)
\]

With a bit of rearranging to (22), it is straightforward to verify that

\[
\text{sign} \left( \frac{\partial \hat{x}_2}{\partial t_2} \right) = \text{sign} \left( \pi_3 [Ax_{2\mu} + BX_{3\mu}] - B + (1 - \pi_3)\frac{\Psi}{x_2} - [Ax_{2\mu} + BX_{3\mu}] \right).
\]

Because our objective is to show different possibilities, we can get some traction by assuming \(\Psi \to 0\), as nothing rules out this possibility. In this case, we have

\[
\text{sign} \left( \frac{\partial \hat{x}_2}{\partial t_2} \right) = \text{sign} \left( \underbrace{\pi_3 [Ax_{2\mu} + BX_{3\mu}] - B - [Ax_{2\mu} + BX_{3\mu}]}_{\leq 0} \right), \quad (26)
\]
where the inequality follows from (25) This expression shows how the sign will depend primarily on the sign and magnitudes of price and income effects.

To focus on what is perhaps the most plausible scenario, let us assume that both \(x_2\) and \(X_3\) are normal (i.e., \(x_2, X_3 \geq 0\)). In this case, it follows that if the two good are net substitutes (i.e., \(A, B > 0\)), the sign of (26) is always negative. It only remains to show the possibility for (26) to be positive without violating any of the conditions. Clearly, a necessary condition is \(Ax_2 + BX_3 < 0\). Continuing to assume both goods are normal, this requires either \(A\) or \(B\) negative. But (24) implies that we cannot satisfy the inequality and have \(A < 0\). Hence \(A > 0\) and \(B < 0\) is a necessary condition, and this means that the two goods are net complements in the case where \(0 > C_{22} > C_{32} (= C_{23}) > C_{33}\). Given that this plausible, we therefore find nothing that rules out the possibility for (26) to be positive.

### A.5 The relationship between the sign of \(\varepsilon_{x_2^{N},t_2}\) and \(\varepsilon_{\hat{x}_2,t_2}\)

We begin with the identity

\[
x_2^N(t_2) = \hat{x}_2(t_2, (n - 1)t_2x_2^N(t_2)).
\]

Differentiating yields

\[
\frac{\partial x_2^N}{\partial t_2} = \frac{\partial \hat{x}_2}{\partial t_2} + \frac{\partial \hat{x}_2}{\partial X_3}(n - 1) \left( x_2^N + t_2 \frac{\partial x_2^N}{\partial t_2} \right),
\]

and rewriting with elasticities implies

\[
\varepsilon_{x_2^{N},t_2} = \frac{\varepsilon_{\hat{x}_2,t_2} + D}{1 - D}, \tag{27}
\]

where \(D \equiv (n - 1) \frac{\partial \hat{x}_3}{\partial X_3} \leq 0\), and the sign follows by the definition \(\frac{\partial \hat{x}_3}{\partial X_3} = \frac{\partial \hat{x}_3}{\partial X_3} - 1\) combined with the assumption in (15). Equation (27) indicates that \(\varepsilon_{x_2^{N},t_2}\) shares the same sign as \(\varepsilon_{\hat{x}_2,t_2}\) in all cases except for one, where \(\varepsilon_{\hat{x}_2,t_2} > 0\) and the numerator of (27) is negative, which is more likely to occur with large \(n\), greater crowding out, or both. More generally, we find that nothing rules out the possibility for \(\varepsilon_{x_2^{N},t_2}\) to take
either sign.

A.6 Derivation of the Relative Magnitudes in Table 2

Let us assume \( n \geq 2 \). The conditions that define the Pareto optimal allocation in (8) can be written as follows:

\[
\frac{U_3^*}{U_1^*} = \frac{1}{p_1 n}, \quad \frac{U_2^*}{U_1^*} = \frac{p_2}{p_1}.
\]

The conditions that define the allocation consistent with the optimal dedicated tax in (16) and (12) can be written in parallel fashion as

\[
\frac{U_3^N}{U_1^N} = \frac{1}{p_1(n + (n - 1)\varepsilon_{N,t_2}}
\]

\[
\frac{U_2^N}{U_1^N} = \frac{p_2 + t_2 \left(1 - \frac{1}{n + (n - 1)\varepsilon_{N,t_2}}\right)}{p_1}
\]

where the second comes from substituting the first into (12) and rearranging. Comparing these conditions, it follows that

\[
\frac{U_3^N}{U_1^N} \geq \frac{U_3^*}{U_1^*} \iff \varepsilon_{N,t_2} \geq 0
\]

and

\[
\frac{U_2^N}{U_1^N} \geq \frac{U_2^*}{U_1^*} \iff \varepsilon_{N,t_2} \geq -1.
\]

Three cases are then useful to consider:

\[
\varepsilon_{N,t_2} \geq 0 \implies \frac{U_3^*}{U_1^*} > \frac{U_3^N}{U_1^N} \text{ and } \frac{U_2^*}{U_1^*} < \frac{U_2^N}{U_1^N}
\]
\( \varepsilon_{x_2^N, t_2} \in (-1, 0) \implies \frac{U_3^*}{U_1^*} < \frac{U_3^N}{U_1^N} \) and \( \frac{U_2^*}{U_1^*} < \frac{U_2^N}{U_1^N} \).

\( \varepsilon_{x_2^N, t_2} \leq -1 \implies \frac{U_3^*}{U_1^*} < \frac{U_3^N}{U_1^N} \) and \( \frac{U_2^*}{U_1^*} \geq \frac{U_2^N}{U_1^N} \).

The results in Table 2 follow by simultaneously needing the satisfy these conditions with strict concavity of the utility function and the budget constraint.
References


