# The Microeconomic Theory of the Rebound Effect and its Welfare Implications[*]

Nathan W. Chan[†] Kenneth Gillingham[‡]

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## Abstract

Economists have long noted that improving energy efficiency could lead to a rebound effect, reducing or possibly even eliminating the energy savings from the efficiency improvement. This paper develops a generalized model to highlight features of the theory of the microeconomic rebound effect that are particularly relevant to empirical economists. We demonstrate when common elasticity identities used for empirical estimation are biased, and how gross complement and substitute relationships govern this bias. Furthermore, we formally derive the welfare implications of the rebound effect to provide clarity for on-going policy debates about the rebound.

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[†] Nathan W. Chan, Colby College, 5246 Mayflower Hill, Waterville, ME 04901, phone: 207-859-5246, e-mail: nwchan@colby.edu.

[‡] Corresponding author: Kenneth Gillingham, Yale University, 195 Prospect Street, New Haven, CT 06511, phone: 203-436-5465, e-mail: kenneth.gillingham@yale.edu.

# 1 Introduction

Economists have long noted that improving energy efficiency could lead to a behavioral response reducing or even eliminating the energy savings from the efficiency improvement. This effect has come to be known in the economics literature as the "rebound effect," suggestive of an initial energy savings and a rebound in energy use from the response. The rebound effect has critical relevance for energy efficiency policymaking, for it underpins whether an energy efficiency policy can be counted on to meet an energy-reduction or emission-reduction target, and it may also lead to secondary benefits and costs from such a policy. Moreover, the rebound effect has motivated a vast literature of empirical work estimating the consumer response to changing energy prices.[1]

This paper develops the theory of the rebound effect from first principles in consumer theory. We proceed with three goals in mind. First, to provide a clean and intuitive treatment of the microeconomic rebound effect in realistic settings with multiple fuels and energy services. Second, to guide empirical economists in the use of common elasticity identities for empirical estimation of the rebound effect. Third, to develop the first formal treatment of the welfare implications of the rebound effect, thereby helping to clarify the discussion in both the academic and policy literature about policy responses to the rebound effect.

The focus of our study is on the "direct" rebound effect, which is the additional fuel use attributable to increased energy service demand when the implicit price of the service declines due to an energy efficiency improvement, and the "indirect" rebound effect, which is the increase in energy consumption from changes in the consumption of other goods and services due to improved energy efficiency in the product of interest.[2] We follow the traditional literature by conceptualizing rebounds as changes in consumer energy consumption, while

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[1] See Gillingham et al. (2013); Sorrell et al. (2009); and IRGC (2013) for reviews.

[2] The indirect rebound is inconsistently defined in the literature. Many authors also include changes in the net embodied energy from the production of all products after the energy efficiency improvement (Sorrell and Dimitropoulos 2008; Azevedo 2014). Others define it as the effect of re-spending any increased income freed-up from the efficiency improvement (not including the compensated substitution effect) (Borenstein 2015). For clarity, we define the indirect rebound as the gross changes in consumption in other goods and services when efficiency changes, but note that embodied energy could be included in our framework.

recognizing that our framework could be readily extended to examine primary energy use and to redefine the effect in terms of greenhouse gas emissions.

Much of the literature discussing the microeconomics of the rebound effect is focused solely on estimating the direct rebound effect. It bases analysis on demand functions, rather than underlying preferences, in a world with a single fuel and single energy service (Sorrell and Dimitropoulos 2008; Small and Van Dender 2007; Frondel and Vance 2013; Gillingham 2011).[3] Others emphasize how the microeconomic rebound effect can be decomposed into the standard substitution and income effects from price theory (Borenstein 2015). Some of the first papers that base analysis on underlying preferences, such as Berkhout et al. (2000) and Binswanger (2001), derive classic relationships between different elasticities relevant to the rebound effect. For example, the fuel price elasticity of fuel or service demand is commonly assumed to be equivalent to the efficiency elasticity of service demand (i.e., the direct rebound effect). This relationship underpins most efforts to estimate the direct rebound effect, which rely on variation in fuel prices and consumption.

Our treatment begins with underlying preferences in a more general setting and derives exactly when these classic relationships hold and when they do not. The closest work to ours is by Hunt and Ryan (2014a,b), who present a similar utility-theoretic model with multiple energy services and input fuels. Hunt and Ryan (2014a) note that estimating the rebound effect in such a setting will be difficult, for we are unlikely to observe expenditure on each energy service. Our treatment shows that there is actually a more fundamental, theoretical issue that can lead to the breakdown of the classic elasticity identities.

With multiple energy services and a single fuel (e.g., electricity) or multiple fuels for a single energy service (e.g., home heating), we find that the fuel price elasticity of fuel or service demand capture fundamentally different implicit price changes than the efficiency elasticity of service demand or service price elasticity of service demand. As such, estimates of the direct rebound effect based on the former two elasticities may be biased. With multiple

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[3] To simplify the exposition, we refer to “fuel” in a broad sense, and describe energy carriers, such as electricity, as fuels.

energy services and a single fuel, we show that the direction and magnitude of this bias is determined by whether the energy services are gross substitutes or gross complements. This theoretical finding has important implications for empirical economists. For example, it suggests that using the common elasticity identities is particularly problematic for studying household electricity use, as home entertainment, computers, and household heating may constitute complementary services that all use electricity. In contrast, these identities may be less problematic for understanding the rebound effect in personal vehicle use in the United States, where there are few substitutes. For multiple fuels and a single service, we show that only the price elasticity for the lowest cost fuel is relevant for the rebound effect.

Surprisingly, there is little discussion of the welfare implications of the microeconomic rebound effect based on welfare economics in the literature. A few papers note that the rebound effect is likely to be welfare improving, barring significant external costs (e.g., Hobbs 1991; Borenstein 2015; Gillingham et al. 2014). These papers contrast with a surprisingly large number of papers in both the economics literature and in the policy realm that discuss policy approaches to "mitigate" the rebound effect (van den Bergh 2011; Ouyang et al. 2010; Herring and Roy 2007; Maxwell et al. 2011; Otto et al. 2014; Gloger 2011). It appears that these ancillary policies to mitigate the effect are based on a cost-effectiveness criterion, with the goal being to meet a particular energy savings or emissions reduction target, rather than maximize social welfare. An alternative explanation is that studies calling for mitigation of the rebound effect implicitly assume that externalities from energy use are very large and are therefore likely to outweigh potential benefits.[4]

Given the serious discussion about ancillary policies to mitigate the rebound effect, a formal exposition of the rebound effect in the context of external costs and welfare is warranted. Our treatment provides clarity on the welfare consequences of the rebound effect and highlights a point that we have not seen discussed in the literature: there is an important difference between externalities arising from fuel usage (e.g., pollution) and externalities

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[4] We thank an anonymous referee for suggesting this latter explanation.

arising from energy service consumption (e.g., traffic congestion). Notably, external costs from energy service provision will necessarily increase because of the rebound effect, while external costs from fuel use may grow or diminish, depending on the magnitude of the rebound effect. We develop formal conditions under which welfare increases or decreases in response to efficiency changes, and we use these conditions to find situations in which energy efficiency policies will improve welfare. We show that energy efficiency improvements are more likely to enhance welfare when the surplus from energy services is high, the cost of the improvement is low, service-based external costs are low, and rebound effects are modest. Less intuitively, we show that when pollution-based external costs are high for other goods and services, the welfare effects depend on gross complement and substitute relationships between the energy services.

This paper focuses entirely on the microeconomics of the rebound. Economists have noted that there is also the possibility of a macroeconomic or economy-wide rebound effect. This can be thought of as the result of changes in relative prices and incomes throughout the global economy with a change in energy efficiency. For example, as explained in Borenstein (2015), Gillingham et al. (2013), and Gillingham and Palmer (2014), there may be a rebound when energy efficiency shifts demand for oil inward, reducing the market price, and increasing the quantity produced. Turner (2013) argues that energy efficiency improvements could crowd out export demands, putting upward pressure on prices through macroeconomic interactions between growth and prices. Other macroeconomic rebound effects due to structural shifts in the economy or induced technological change are also possible (Barker et al. 2007; Wei 2010). These effects are outside the scope of this paper but may be important.

The remainder of the paper is organized as follows. The next section presents a general utility-theoretic model to fix ideas. Section 3 illustrates how a simplification of this model produces common findings in the literature. Section 4 examines subcases with multiple energy services or multiple fuels and demonstrates when common intuition no longer holds. Section 5 explores the welfare implications of the rebound effect. Finally, Section 6 concludes.

# 2 General Model

We begin by setting up a general model of energy service and fuel demand to set the stage for our analysis. We allow fuels to be used to generate multiple energy services, just as occurs in reality. For example, electricity can be used for everything from heating to cooking to powering electric devices. We also allow a given energy service to be obtained through different fuels, just as home heating can be provided by electric heat or gas heat. We focus on a static setting that characterizes the features of the consumer decision relevant to the rebound effect.

The foundation of the model is as follows. Consumer utility is defined over energy services $s_i$ and a composite non-energy numeraire $x$. Of course, nearly all goods require some energy in consumption and production, but to simplify, we call any good that requires very low energy input as part of the 'non-energy numeriare good.' We consider two energy services, $i = 1, 2$.[5] Energy service $i$ is obtained through the consumption of two fuels $j = 1, 2$ that are priced at $p_j$. The price of the numeraire good $x$ is normalized to unity. Fuel $j$ is converted into service $i$ using a technology with fuel efficiency $\eta_{ij}$, with the corresponding consumption of fuel $j$ for service $i$ being given by $f_{ij}$.

The consumer has income $w$ that he spends on fuels and the numeraire good. The consumer problem is given by

$$\begin{aligned} \max_{x,s_1,s_2} \quad & U(x, s_1, s_2) \\ \text{subject to} \quad & s_1 = \eta_{11} f_{11} + \eta_{12} f_{12} \\ & s_2 = \eta_{21} f_{21} + \eta_{22} f_{22} \\ & w = x + p_1 \left(f_{11} + f_{21}\right) + p_2 \left(f_{12} + f_{22}\right). \end{aligned}$$

This general framework subsumes several frameworks in the existing literature and allows us to examine the generalizability of existing results. In order to focus entirely on new

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[5] One way to interpret $s_2$ is as a composite energy service that captures all energy services besides $s_1$.

insights on the rebound effect, this framework abstracts away from costly investments in energy efficiency,[6] any dynamics of the decision process, and any behavioral anomalies in the decision process.

We begin by deriving some common results under the assumption that there is a one-to-one correspondence between energy services and fuels. Then we will generalize and show the conditions under which these common results fail.

## 3 Insights from the Classic Model

We begin with a simplifying assumption that is employed in nearly all theoretical discussions of the rebound effect. With this assumption in place, we will reveal how many of the basic conclusions from previous work on the rebound effect arise. We will later relax this assumption in Section 4 to provide new results showing how these conclusions can be overturned in a more general setting.

**Assumption 1:** There is a one-to-one correspondence (bijection) between energy services and fuels, so that each fuel is used for a single energy service, and each energy service can only be obtained by a single fuel.

In this case, $f_{12} = f_{21} = 0$, and the model simplifies to

$$\begin{aligned} \max_{x,s_1,s_2} \quad & U(x, s_1, s_2) \\ \text{subject to} \quad & s_1 = \eta_{11} f_{11} \\ & s_2 = \eta_{22} f_{22} \\ & w = x + p_1 f_{11} + p_2 f_{22}. \end{aligned}$$

The solution to the consumer maximization problem yields the demand for energy service $i$, denoted $s_i^* (\eta_{11}, \eta_{22}, p_1, p_2, w)$. This can be conveniently rewritten as $s_i^*(\pi_1, \pi_2, w)$, where

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[6] Borenstein (2015) shows that such investments reduce the magnitude of rebound effects by decreasing income.

$\pi_i = \frac{p_i}{\eta_{ii}}$ is the implicit price of the energy service $i$.

We can use $s_i^*$ to obtain the demand for fuel consumption $i$ as a function of the vector of implicit prices $\boldsymbol{\pi}$ and income $w$, according to

$$f_{ii}^* (\boldsymbol{\pi}, w) = s_i^* (\boldsymbol{\pi}, w) / \eta_{ii}, \tag{1}$$

with asterisks to indicate that these functions arise as solutions to the utility maximization.

For the rest of the paper, we will consider a case where energy service 1 receives an energy efficiency improvement. With an improvement in $\eta_{11}$ we obtain the following comparative statics, which provide several insights on the direct and indirect rebound effect:

$$\frac{\partial s_1^*}{\partial \eta_{11}} = -\frac{p_1}{(\eta_{11})^2} \frac{\partial s_1^*}{\partial \pi_1} \tag{2}$$

$$\frac{\partial f_{11}^*}{\partial \eta_{11}} = -\frac{1}{(\eta_{11})^2} \left( \frac{p_1}{\eta_{11}} \frac{\partial s_1^*}{\partial \pi_1} + s_1^* \right) \tag{3}$$

$$\frac{\partial s_2^*}{\partial \eta_{11}} = \eta_2 \frac{\partial f_{22}^*}{\partial \eta_{11}} = -\frac{p_1}{(\eta_{11})^2} \frac{\partial s_2^*}{\partial \pi_1}. \tag{4}$$

These comparative statics show how changes in energy service and fuel demand with an increase in $\eta_{11}$ can be mapped back to changes in energy service demand with a decrease in the implicit price of the energy service. The following discussion will rely on each of these.

### 3.1 Direct rebound

The direct rebound effect is the fuel consumption from the additional use of energy service 1 resulting from an improvement in $\eta_{11}$ due to the decrease in the implicit price of usage $\pi_1$. With an improvement in $\eta_{11}$, there will be a substitution effect and income effect, both of which increase usage $s_1^*$ and accordingly influence fuel usage $f_{11}^*$. The direct rebound effect is often defined in terms of elasticities. Let $\varepsilon_{a,b}$ denote the elasticity of demand for $a$ with respect to $b$. Then, the direct rebound effect is $\varepsilon_{f_{11},\eta_{11}} + 1$ or equivalently $\varepsilon_{s_1,\eta_{11}}$.

To illustrate, suppose there there is no change in $s_1$ demanded when efficiency changes.

In this case, $\varepsilon_{s_1,\eta_{11}} = 0$, indicating that the direct rebound effect is zero. Equivalently, $\varepsilon_{f_{11},\eta_{11}} = -1$, as the entire improvement in efficiency will be realized as a decrease in fuel consumption. In contrast consider a case where all of the fuel savings from the energy efficiency improvement are taken-back by fuel use from increased service consumption. In this case, we have $\varepsilon_{s_1,\eta_{11}} = 1$, which represents a 100% direct rebound effect. We would also have $\varepsilon_{f_{11},\eta_{11}} = 0$, as there are no net fuel savings from the efficiency improvement. The case of a greater than 100% rebound is popularly known as "backfire."

Several important elasticity relationships commonly used in the literature immediately emerge from our framework:

$$\varepsilon_{s_1,\eta_{11}} = \varepsilon_{f_{11},\eta_{11}} + 1 = -\varepsilon_{f_{11},p_1} = -\varepsilon_{s_1,\pi_1} = -\varepsilon_{s_1,p_1}. \tag{5}$$

All of these follow directly from Expressions 2, 3 and 4 and are proven elsewhere (Sorrell and Dimitropoulos 2008; Gillingham 2011; Thomas and Azevedo 2013a), so we do not prove them here. They make the standard neoclassical assumptions that raising energy efficiency and decreasing energy prices have the same impact on service demand, that the response to increases and decreases in fuel prices is symmetric, and that energy prices do not depend upon efficiency (Sorrell and Dimitropoulos 2008; Frondel and Vance 2013). They also assume no capital costs from the energy efficiency investment (Sorrell and Dimitropoulos 2008).

These identities imply that empirical economists can estimate $\varepsilon_{s_1,p_1}$ or even $\varepsilon_{f_{11},p_1}$ to quantify the direct rebound effect. This is commonly done, since it only requires plausibly exogenous variation in fuel prices, rather than energy efficiency, which is much more difficult to obtain. Our framework will show the danger in using these identities when we relax Assumption 1.

## 3.2 Indirect rebound

The indirect rebound effect is defined for our purposes as the increased fuel use from the consumption of other energy services when $\eta_{11}$ increases. This effect is due to the income and substitution effects on all other energy services with the increase in $\eta_{11}$. In terms of elasticities, the indirect rebound can be defined as $\varepsilon_{f_{22},\eta_{11}}$ or equivalently $\varepsilon_{s_2,\eta_{11}} = -\varepsilon_{s_2,\pi_1}$. From this, it is clear that $s_2^*$ will increase (decrease) with an increase in $\eta_{11}$ if energy service 2 is a gross complement (substitute) for energy service 1.[7] Notably, the use of other energy services and fuels may increase or decrease when $\eta_{11}$ improves. While the importance of complements and substitutes has been loosely alluded to before (e.g., Berkhout et al. (2000) and Binswanger (2001)), our framework provides precision and clarity in demonstrating that the indirect rebound is dictated by *gross*, rather than net, complement and substitute relationships, a point also described in a working paper by Sorrell (2012).

The indirect rebound effect has received considerably less attention in the empirical literature than the direct rebound, in part because of inherent difficulties in estimating cross-price elasticities. Not only is it challenging to estimate a comprehensive model of household energy demand complete with all relevant energy sources, but it is even more difficult to embed energy service consumption, which is often not observable to analysts, into such a model. Thus, it is common to ignore the indirect rebound altogether or equate the indirect rebound effect with the fraction of the consumer budget spent on energy-using services (e.g., see the discussion in Greening et al. (2000) or Borenstein (2015)). However, since the indirect rebound effect depends on the change in fuel expenditure on the margin, estimates of the indirect rebound effect based on the average budget share of energy-using goods are likely biased and could even have the wrong sign.

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[7] Here we see a useful parallel to the literature on impure public goods. When an impure public good is improved so as to decrease the implicit price of a private characteristic, demand for the public characteristic will rise or fall depending on whether the public characteristic is a gross complement or substitute, respectively, for the private characteristic (Kotchen 2005; Chan and Kotchen 2014).

## 3.3 Combined microeconomic rebound

Thus far, we have shed light on the factors that influence the magnitude and direction of the direct and indirect rebound effect. But what is the relationship between the direct and indirect rebound effect, and what is the net impact on energy usage? Is it possible for both types of rebounds to cause additional fuel use after an efficiency improvement? The following discussion shows that the answer to this depends on whether the consumer derives utility from only energy services or both energy services and non-energy services.

### 3.3.1 Case 1: Only energy services

Consider a further simplification to the model where non-energy services do not enter into the utility function. Then, utility is defined only over energy services: $U(s_1, s_2)$. While this may seem like an extreme assumption, it is one that is often adopted, implicitly or explicitly, in previous theoretical explorations of the rebound (e.g., Berkhout et al. 2000; Binswanger 2001). We will show how adopting such a restrictive assumption yields conclusions that turn out to be incorrect when non-energy services are included.

**Remark 1.** *When utility is defined over energy services $s_1$ and $s_2$ only, the direct and indirect rebound effects are countervailing. That is, a large direct rebound will be accompanied by a small (or negative) indirect rebound, and a large indirect rebound will be accompanied by a small direct rebound.*

Formally, these results arise from the fact that comparative static of the budget constraint is such that $p_1 \frac{\partial f_{11}^*}{\partial \eta_{11}} + p_2 \frac{\partial f_{22}^*}{\partial \eta_{11}} = 0$, and therefore, $\frac{\partial f_{11}^*}{\partial \eta_{11}}$ and $\frac{\partial f_{22}^*}{\partial \eta_{11}}$ must have opposite signs. This can also be seen graphically in Figure 1. Figure 1 shows how an increase in $\eta_1$ to $\eta_1'$ decreases the implicit price from $\pi_1 = \frac{p_1}{\eta_{11}}$ to $\pi_1' = \frac{p_1}{\eta_{11}'}$, which pivots the budget constraint upward by $\frac{\eta_{11}'}{\eta_{11}}$. That is, with an x% improvement in efficiency, the consumer can purchase x% more $s_1$ than before; thus, each point along the new budget constraint is x% higher than the old budget constraint.

[ INSERT FIGURE 1 HERE ]

Suppose point A is the initial consumption bundle. Assuming an ordinary good, the new optimal consumption bundle must lie along the new budget constraint between points B and D, according to the law of demand. Of course, if energy service 1 is a Giffen or Veblen good, then it is possible to observe a consumption bundle between points D and E. Point D represents an extreme case in which there is no direct rebound, which is only possible if both the substitution and income effects are zero or there is a positive substitution effect exactly offset by a negative income effect, as in the case of an inferior good. In this extreme case, the consumption of energy service 1 remains exactly the same as at point A (i.e., there is no behavioral response and the engineering estimate of energy savings from $s_1$ is correct). However, in this case, all monetary savings are re-spent on $s_2$, so that there is a large indirect rebound.

Meanwhile, backfire occurs at any point between B and C. Point C represents a situation where consumption of $s_1$ increases enough to exactly offset the energy savings predicted by the engineering estimate.[8] This is where the rebound effect is 100%, and there is no indirect rebound. To the left of point C on the dotted budget constraint, consumption of $s_1$ increases by so much that it more than offsets the purported energy savings, and therefore energy use from $s_1$ actually increases. At the same time, energy use from $s_2$ decreases, so there is a negative indirect rebound.

Equations (2), (4), and (5) help formalize these intuitions. The Slutsky decompositions of (2) and (4) reveal that the comparative statics for efficiency are composed of substitution and income effects. For example, we can rewrite (2) as $\frac{\partial s_1^*}{\partial \eta_{11}} = -\frac{p_1}{(\eta_{11})^2}\left(\frac{\partial s_1^h}{\partial p_1} - \frac{\partial s_1^*}{\partial w} s_1^*\right)$, where the superscript $h$ denotes the Hicksian (compensated) demand. Under standard assumptions, $\frac{\partial s_1^h}{\partial p_1}$ is weakly negative and bounded above by zero (ordinary good) and $\frac{\partial s_1^*}{\partial w}$ is weakly positive

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[8] To see this, let $s_1^A$ be $s_1$ consumed at point A and let $s_1^C$ be $s_1$ consumed at point C. Note that $s_1^C = \frac{\eta_{11}'}{\eta_{11}} s_1^A$. Moreover, energy consumption for $s_1$ was $\frac{s_1^A}{\eta_{11}}$ before the efficiency improvement and $\frac{s_1^C}{\eta_{11}'}$ after the improvement. Plugging in the identity for $s_1^C$, it follows that energy consumption is the same at points A and C.

and bounded below by zero (normal good). Therefore, $\frac{\partial s_1^*}{\partial \eta_{11}}$ is weakly positive. Referring back to the graph, point D respresents the boundary case in which $\varepsilon_{s_1,\eta_{11}} = 0$. Meanwhile, point C represents the point at which $\varepsilon_{s_1,\eta_{11}} = 1$ (see equation (5)), signifying that the percent increase in demand for $s_1$ is exactly equal to the percent increase in $\eta_{11}$.

These results suggest that countervailing direct and indirect rebound effects would serve to moderate the combined rebound effect. The intuition behind this finding is that if more income is spent on $s_1$, less is available for $s_2$ and vice versa. Furthermore, countervailing direct and indirect effects imply that if we have "backfire" from the direct rebound effect, there must be a corresponding decrease in fuel use from other energy services. The idea that there may be such countervailing effects is described in Thomas and Azevedo (2013b), and is consistent with discussions in Borenstein (2015) and Gillingham et al. (2014).

### 3.3.2 Case 2: Energy services and non-energy services

In reality, there may be goods and services that are desirable and costly but require a negligible amount of energy. For example, books, magazines, and board games require relatively little energy to consume and may have only a small amount of embodied energy from production. Recall that we denote services that require little energy the 'non-energy numeriare' $x$. It turns out that in the presence of such non-energy services, the result in Remark 1 no longer holds.

**Remark 2.** *When utility is defined over energy services $s_1$ and $s_2$ and one or more non-energy services $x$, the direct and indirect rebound effects need not countervail.*

The relationship between the direct and indirect rebound effect is mediated by the complement/substitute relationship between the non-energy numeraire good $x$ and $s_1$. Here, the comparative static of the budget constraint is $\frac{\partial x^*}{\partial \eta_{11}} + p_1 \frac{\partial f_{11}^*}{\partial \eta_{11}} + p_2 \frac{\partial f_{22}^*}{\partial \eta_{11}} = 0$. If $x$ is a gross substitute for $s_1$, then expenditure on $x$ will decrease, making more income available for both energy services. As such, it becomes possible for demands for both fuels $f_{11}^*$ and $f_{22}^*$ to increase, indicating that direct rebound backfire could be accompanied by a concurrent increase in fuel usage for other energy services. For further intuition, consider Figure 1 again. We can think of the demand for $x$ as a third axis. As demand for $x$ decreases, the effective budget constraint in $(s_1, s_2)-$space expands, making possible a scenario in which both $f^*_{11}$ and $f^*_{22}$ increase.

This point may appear intuitive and is mentioned in Sorrell (2012), but surprisingly is not more broadly known. It also has clear implications for empirical estimation and policy analysis. Namely, we cannot simply assume that if the direct rebound effect is large that there will be a countervailing indirect rebound effect that will lessen its impact. If we have reason to believe that $x$ is a gross substitute for $s_1$, then it is possible for increases in fuel use for both the good in question and other goods. In section 5, we will return to these results and show their importance for estimating welfare changes.

## 4 Multiple Energy Services and Fuels

In reality, there is not a straightforward one-to-one correspondence between energy services and fuels. For example, electricity is used for many energy services, including heating, lighting, and refrigeration. Likewise, an energy service like heating can be provided using variety of fuels, such as electricity, natural gas, or heating oil, and many households have the ability to substitute between these (e.g., by using an electric space heater instead of central oil or gas heat).[9] Hunt and Ryan (2014a,b) are the only other papers in the literature we are aware of to recognize this more general case. In contrast to the treatment in these papers, we differentiate between two cases: (1) one fuel for many services and (2) many fuels for one service. Interestingly, we find that each case yields different insights and complications for empirical estimation of the rebound effect.

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[9] Strictly speaking, electricity is an energy carrier rather than a fuel. To maintain consistency and to minimize jargon, we will abuse vocabulary slightly and continue to refer to it as a fuel.

## 4.1 More energy services than fuels

Modify Assumption 1 to allow a single fuel, such as electricity, to provide multiple energy services.

**Assumption 1a:** Each fuel type may be used as an input for multiple energy services.

Assumption 1a generalizes the treatment of the consumer problem in Section 3, but still restricts it from the fully general problem in Section 2. We will again work through the case of two energy services. Consider one fuel. Following the notation in our general model in Section 2, we denote the demand for fuel used for $s_1^*$ as $f_{11}^*$, and the demand for fuel used for $s_2^*$ as $f_{21}^*$.

Now the consumer problem is:

$$\begin{aligned} \max_{x,s_1,s_2} \quad & U(x, s_1, s_2) \\ \text{subject to} \quad & s_1 = \eta_{11} f_{11} \\ & s_2 = \eta_{21} f_{21} \\ & w = x + p_1(f_{11} + f_{21}). \end{aligned}$$

Maximizing, we obtain a solution for the demand for energy service $i$ of the form

$$s_i^* \left( \frac{p_1}{\eta_{11}}, \frac{p_1}{\eta_{21}}, w \right) = s_i^* \left( \pi_{11}, \pi_{21}, w \right).$$

As before, the solution for $s_i^*$ implies the solution for $f_{i1}^*$, according to

$$f_{i1}^*(\pi_{11}, \pi_{21}, w) = s_i^* \left( \pi_{11}, \pi_{21}, w \right) / \eta_{i1}.$$

As discussed in Section 3, it is common practice to use either the fuel price elasticity of energy service demand or the fuel price elasticity of fuel demand as estimates of the direct rebound effect. However, under Assumption 1a, this is no longer a valid measure of the rebound effect. Hunt and Ryan (2014a) note that data limitations prevent us from observing the expenditure on each service; instead, we can only observe the total expenditure on the shared fuel, thus presenting challenges for estimation. While this is an important point, our analysis shows that there is more than a problem of limited data. Even if we could observe service-specific expenditures, the classic elasticity relationship that is used to identify the rebound effect no longer holds.

**Proposition 1.** *When a fuel $f_{i1}$ can be used as an input for multiple energy services $s_1$ and $s_2$, we have $\varepsilon_{s_1,p_1} = \varepsilon_{f_{11},p_1} = \varepsilon_{s_1,\pi_{11}} + \varepsilon_{s_1,\pi_{21}} \neq -\left(\varepsilon_{f_{11},\eta_{11}} + 1\right) = \varepsilon_{s_1,\pi_{11}} = -\varepsilon_{s_1,\eta_{11}}$.*

*Proof.* The first equality holds from (5). Recall that fuel demands are defined as $f^*_{i1}\left(\pi_{11}, \pi_{21}, w\right) = \frac{s^*_i(\pi_{11},\pi_{21},w)}{\eta_{i1}}$ and implicit prices as $\pi_{i1} = \frac{p_1}{\eta_{i1}}$. We have $\frac{\partial f^*_{11}}{\partial p_1} = \frac{1}{\eta_{11}}\left(\frac{\partial s^*_1}{\partial \pi_{11}}\frac{\partial \pi_{11}}{\partial p_1} + \frac{\partial s^*_1}{\partial \pi_{21}}\frac{\partial \pi_{21}}{\partial p_1}\right) = \frac{1}{\eta_{11}}\left(\frac{1}{\eta_{11}}\frac{\partial s^*_1}{\partial \pi_{11}} + \frac{1}{\eta_{21}}\frac{\partial s^*_1}{\partial \pi_{21}}\right)$, where the expressions on the right hand side follow from differentiating $s^*_1$. Multiplying the latter expression by $\frac{p_1}{f_{11}}$ on both sides and substituting identities for $\pi_{i1}$ and $f^*_{11}$, we obtain $\varepsilon_{f_{11},p_1} = \frac{\partial s^*_1}{\partial \pi_{11}}\frac{\pi_{11}}{s^*_1} + \frac{\partial s^*_1}{\partial \pi_{21}}\frac{\pi_{21}}{s^*_1} = \varepsilon_{s_1,\pi_{11}} + \varepsilon_{s_1,\pi_{21}}$. Meanwhile, $\frac{\partial f^*_{11}}{\partial \eta_{11}} = -\frac{s^*_1}{\eta^2_{11}}\left(1 + \frac{\pi_{11}}{s_1}\frac{\partial s^*_1}{\partial \pi_{11}}\right)$, so $-\left(\varepsilon_{f_{11},\eta_{11}} + 1\right) = \varepsilon_{s_1,\pi_{11}}$. □

This proposition has significant implications for empirical estimation, as $\varepsilon_{s_1,p_1}$ and $\varepsilon_{f_{11},p_1}$ are biased estimates of $-\left(\varepsilon_{f_{11},\eta_{11}} + 1\right)$. Recall from section 3 that the direct rebound effect is defined as $\left(\varepsilon_{f_{11},\eta_{11}} + 1\right)$, so this result elucidates the deeper theoretical bias from using $\varepsilon_{s_1,p_1}$ and $\varepsilon_{f_{11},p_1}$ to estimate the direct rebound effect. Notably, our analysis identifies this bias as $\varepsilon_{s_1,\pi_{21}}$, allowing us to predict the direction of this bias and perhaps even quantify it.

**Corollary 2.** *$-\varepsilon_{s_1,p_1}$ and $-\varepsilon_{f_{11},p_1}$ will overestimate (underestimate) the magnitude of the direct rebound effect $\left(\varepsilon_{f_{11},\eta_{11}} + 1\right)$ when $s_1$ is a gross complement (substitute) for $s_2$.*

For intuition, consider household air conditioning (AC) and in-home electronics, which we assume are complements. Further, consider a doubling in AC efficiency and an associated increase in usage from the direct rebound effect of 10%. An attempt to estimate such a rebound effect using variation in fuel prices will be confounded by the fact that electricity is used to power both air conditioning and in-home electronics. Why? The price elasticity of electricity demand for air conditioning is measured in a setting in which overall electricity prices have changed, so the implicit price of a complement (in-home electronics usage) has also changed. In contrast, when an AC becomes more efficient, air conditioning becomes cheaper, but the implicit price of in-home electronics usage remains constant.

Numerically, this could be important. Consider a decrease in electricity prices that would yield the same implicit price change as the doubling of AC efficiency. Such a price decrease would increase AC usage by 10% due to the lower price of cooling. However, there will be a further increase in AC usage because the price of complements such as television and lighting has also gone down. Suppose this increase is 3%. Then the magnitude of our proxy, the estimated fuel price elasticity, will exceed that of the true direct rebound effect by 3 percentage points. Meanwhile, the converse is true for substitutes, which would result in an underestimate of the true rebound effect.

We are the first to identify and characterize this bias in common empirical estimates of the rebound effect. Notably, our analysis calls into question the practice of using fuel price elasticities to estimate the rebound effect when multiple services are obtained from the same fuel. That being said, there may be some hope for estimating the rebound through fuel price changes. When the cross-price elasticity between energy services that use the same fuel is small, these confounding interactions are greatly diminished, and the price elasticity of fuel demand is a reasonable proxy for the rebound effect. Thus, the common approach for estimating the rebound effect is likely to be valid when analyzing personal vehicle driving behavior in the United States, where gasoline-consuming complements or substitutes for driving, such as buses, are relatively uncommon. On the other hand, as our example suggests, there may be important complementarities (or substitutabilities) across energy services that use electricity in the home, such as appliances and lighting. In such cases, our results indicate that a better measure of the direct rebound effect would be the service price elasticity $\varepsilon_{s_1,\pi_{11}}$, which may be more difficult to measure, but would not be biased.

## 4.2 Multiple fuels for each energy service

In many situations, we have multiple fuels that can provide the same energy service. For example, household heating can be obtained from electricity, gas, oil, or firewood, and consumers can drive gasoline or diesel vehicles. We will again adjust Assumption 1.

**Assumption 1b:** Multiple fuels can be used to provide a single energy service.

For our purposes, it suffices to consider a case where utility is obtained from a single energy service $s_1$ and a numeraire non-energy service $x$. We will also confine our attention to two fuels $f_{11}$ and $f_{12}$, which are both capable of providing $s_1$. We assume that regardless of fuel, the energy service has the same quality and that there are no adjustment costs from switching between fuels. The efficiency of producing $s_1$ with fuel $j$ is given by $\eta_{1j}$, as in Section 2. The consumer problem is as follows.

$$\begin{aligned} \max_{x,s_1,s_2} \quad & U(x,s_1) \\ \text{subject to} \quad & s_1 = \eta_{11} f_{11} + \eta_{12} f_{12} \\ & w = x + p_1 f_{11} + p_2 f_{12}. \end{aligned}$$

Maximizing, we obtain demand for the energy service of the form

$$s_1^* \left(\eta_{11}, \eta_{12}, p_1, p_2, w\right) = s_1^* \left(\frac{p_1}{\eta_{11}}, \frac{p_2}{\eta_{12}}, w\right).$$

In this setting, the consumer will choose the least expensive fuel option to provide the energy service. Let $j^*$ be the chosen fuel. The following proposition formalizes this behavior.

**Proposition 3.** *Under Assumption 1b, the consumer will choose* $f_{1j}^*$ *such that* $f_{1j}^* \geq 0$ *for* $j^*$ *and* $f_{1j}^* = 0$ *for* $j \neq j^*$. *The optimal* $j^*$ *is chosen such that* $\frac{p_{j^*}}{\eta_{1j^*}} \in \min\{\frac{p_1}{\eta_{11}}, \frac{p_2}{\eta_{12}}\}$.

*Proof.* By the constraint $s_1 = \eta_{11} f_{11} + \eta_{12} f_{12}$, the $f_{1j}$'s enter the utility function as perfect substitutes. This can be seen easily by rewriting the utility function as $U(x, \eta_{11} f_{11} + \eta_{12} f_{12})$.

Thus, the maximizing consumer will choose $j^*$ that yields the lowest implicit price $\frac{p_{j^*}}{\eta_{1j^*}}$. All other fuels face a corner solution with $f^*_{1j} = 0$. $\square$

One result of this proposition is that the common equivalence of elasticities in (5) no longer holds for $j \neq j^*$. Specifically, since $f^*_{1j} = 0$ for $j \neq j^*$, as long as fuel $j$ remains a corner solution, $\varepsilon_{f_{1j},p_j} = 0$, it is meaningless to use estimated elasticities as estimates of the direct rebound effect. For fuel $j^*$, the insights of our simpler model in section 3 continue to apply.

The above proposition also shows that if price or efficiency changes for fuel $j'$ are sufficiently large such that $\frac{p_{j'}}{\eta_{1j'}} < \frac{p_{j^*}}{\eta_{1j^*}}$, the consumer will switch entirely from fuel $j^*$ to $j'$. While the potential for such fuel switching is intuitive, we are the first to show how it relates to the rebound effect. Even if there are adjustment costs to fuel switching, our basic result would still hold, only the relevant fuel for the rebound effect would be the lowest cost fuel after accounting for the adjustment costs. Moreover, this framework could be extended to allow for different technologies that provide distinct, but highly substitutable services (e.g., electric space heaters and central gas-based heating). In that case, the relevant fuel would be the lowest cost fuel accounting for the differences in utility from the different energy services.

This switching result has important implications for energy efficiency policy. For example, a driver of a gasoline vehicle may switch to a diesel vehicle if there is a major improvement in diesel engine efficiency. If the diesel vehicle is more polluting (e.g., it may have higher emissions of particulate matter), then the efficiency improvement may exacerbate local air quality issues. Similarly, innovations in gas furnaces may lead households to switch from wood furnaces, which are fueled by biomass, to natural gas instead. While fuel-switching is commonly studied in development economics (Heltberg 2004, 2005; Gupta and Köhlin 2006; Chambwera and Folmer 2007), it has received little attention in studies of household energy demand in developed countries. Our analysis suggests that empirical researchers may be well-advised to carefully consider fuel-switching behavior in studies of household energy demand and the rebound effect, especially when there are undesirable consequences of such fuel-switching behavior. The next section explores the relationship between the rebound effect and externalities in greater depth in order to shed light on what ultimately matters: social welfare.

## 5 Externalities and Welfare

In order to understand the social welfare implications of the rebound effect, it is critical to explore how the microeconomic rebound effect relates to externalities. This exposition is the first formal treatment of externalities and welfare in the context of the rebound effect. We again restrict our analysis to consumers, effectively assuming perfectly competitive markets in a long-run equilibrium, so that there is no producer surplus. We will answer two distinct, but related questions in the following analysis: (1) When is an energy efficiency improvement beneficial for social welfare? and (2) When is the rebound effect itself beneficial for social welfare?

The importance of the first question is clear, as it speaks to whether or not an energy efficiency policy should be implemented. However, the second question warrants some explanation. There has been extensive discussion of the need to "mitigate" the rebound effect. There are frequent calls, both in the academic literature (van den Bergh 2011; Ouyang et al. 2010; Herring and Roy 2007; Maxwell et al. 2011) and in policy circles (Otto et al. 2014; Gloger 2011), for the rebound effect to be mitigated through voluntary conservation behavior, price instruments, or quantity instruments. Often, these entreaties take as given that the rebound effect is undesirable. Several authors have described why such a perspective may be shortsighted, as consumers also obtain consumer surplus from the microeconomic rebound effect (Hobbs 1991; Saunders 1992; Fong 2011; Saunders and Tsao 2012; Gillingham and Palmer 2014; Borenstein 2015). This section formally elucidates the conditions under which the microeconomic rebound effect is beneficial or harmful in order to offer greater clarity from both a scientific and a policy perspective. Many of our findings accord with economic intuition, but the precision of our formal treatment provides further insight than can be expected from intuition alone.

Recall that our general utility function is $U(x, s_1, s_2)$. For simplicity of exposition, assume a money-metric utility function that is linearly separable in the numeraire: $U(x, s_1, s_2) = x + u(s_1, s_2)$. Moreover, suppose that there are external costs accruing from the usage of fuels and services, and these external costs are linearly aggregable across a population of $k$ identical consumers. Then we can represent social welfare ($SW$) using the representative agent's utility while accounting for damages from externalities:

$$SW = x + u(s_1, s_2) - EC,$$

where total external costs are given by $EC = k(e_1 f_{11} + e_2 f_{22} + c_1 s_1 + c_2 s_2)$. Here, $e_i$ represents the marginal external costs per unit of fuel (e.g., from air pollution), while $c_i$ is the marginal external costs per unit of service (e.g., from traffic congestion or noise pollution from air conditioning). Following an exogenous efficiency improvement from $\eta_{11}$ to $\eta'_{11}$, the change in social welfare is captured by $\int_{\eta_{11}}^{\eta'_{11}} \frac{\partial SW}{\partial \eta_{11}} d\eta_{11}$, where

$$\frac{\partial SW}{\partial \eta_{11}} = \frac{\partial SW}{\partial x}\frac{\partial x^*}{\partial \eta_{11}} + \frac{\partial SW}{\partial s_1}\frac{\partial s_1^*}{\partial \eta_{11}} + \frac{\partial SW}{\partial s_2}\frac{\partial s_2^*}{\partial \eta_{11}} - \frac{\partial SW}{\partial EC}\frac{\partial EC}{\partial \eta_{11}}.$$

Substituting from the budget constraint for the numeraire ($x = w - p_1 f_{11} - p_2 f_{22}$), and evaluating each term, we have:

$$\frac{\partial SW}{\partial \eta_{11}} = -p_1\frac{\partial f_{11}^*}{\partial \eta_{11}} - p_2\frac{\partial f_{22}^*}{\partial \eta_{11}} + \frac{\partial u}{\partial s_1}\frac{\partial s_1^*}{\partial \eta_{11}} + \frac{\partial u}{\partial s_2}\frac{\partial s_2^*}{\partial \eta_{11}} - k\left(e_1\frac{\partial f_{11}^*}{\partial \eta_{11}} + e_2\frac{\partial f_{22}^*}{\partial \eta_{11}} + c_1\frac{\partial s_1^*}{\partial \eta_{11}} + c_2\frac{\partial s_2^*}{\partial \eta_{11}}\right).$$

This expression assumes a costless improvement in energy efficiency for simplicity. If there is a cost to the energy efficiency improvement, the net change in social welfare on the margin would be the difference between the cost and this expression.

We set the stage with the following straightforward proposition.

**Proposition 4.** *Absent externalities ($e_1 = e_2 = c_1 = c_2 = 0$), an exogenous costless increase in $\eta_1$ necessarily improves social welfare.*

*Proof.* An increase in $\eta_{11}$ decreases $\pi_1$, expanding the consumer's choice set and increasing consumer surplus (Le Chatelier's principle). □

This proposition holds regardless of the magnitude of the rebound effect, which highlights that the rebound effect itself is welfare-improving in the absence of externalities. Of course, when there are external costs, the welfare gains from the expanded choice set can be offset by the externalities. Thus, it is unclear whether an efficiency improvement will increase or decrease social welfare.

**Corollary 5.** *When externality costs are present, an exogenous costless increase in $\eta_{11}$ may increase or decrease social welfare.*

We can decompose the welfare change due to energy efficiency improvements into component parts. A full derivation of this result is available in the Appendix.

$$\begin{aligned}\frac{\partial SW}{\partial \eta_{11}} = \quad & \underbrace{\frac{f_{11}^*}{\eta_{11}}(p_1+ke_1)}_{\substack{\text{Averted social}\\ \text{costs with no rebound}}} \quad + \quad \underbrace{\frac{\varepsilon_{s_1,\pi_1}}{\eta_{11}}\left(f_{11}^*\left(p_1+ke_1\right)+s_1^*\left(kc_1-\frac{\partial u}{\partial s_1}\right)\right)}_{\substack{\text{Net Benefits (Costs)}\\ \text{of Direct Rebound}}} \\ & \quad + \quad \underbrace{\frac{\varepsilon_{s_2,\pi_1}}{\eta_{11}}\left(f_{22}^*\left(p_2+ke_2\right)+s_2^*\left(kc_2-\frac{\partial u}{\partial s_2}\right)\right)}_{\substack{\text{Net Benefits (Costs)}\\ \text{of Indirect Rebound}}} \end{aligned} \tag{6}$$

The first term can be interpreted as the averted social costs attributable to the efficiency improvement if no microeconomic rebound effect were to occur. From (3), the change in fuel use attributable to a change in efficiency is given as $\frac{\partial f_{11}^*}{\partial \eta_{11}} = -\frac{f_{11}^*}{\eta_{11}}(1+\varepsilon_{s_1,\pi_1})$. Assuming no direct rebound ($\varepsilon_{s_1,\pi_1} = 0$), the fuel change at the margin is $-\frac{f_{11}}{\eta_{11}}$. Meanwhile, $p_1 + ke_1$

represents the marginal social cost of fuel consumption, which includes the price of fuel as well as the marginal external costs of fuel scaled by the number of people in the market. Therefore, in a case with no direct rebound, the product $\frac{f_{11}^*}{\eta_{11}}(p_1 + ke_1)$ captures the avoided social costs associated with the decrease in fuel usage.

The second cluster of terms represents the net benefits (costs) accruing from direct rebound, where $\varepsilon_{s_1,\pi_1}$ captures the magnitude of the direct rebound and the terms in brackets represent the marginal costs of additional fuel consumption and the marginal costs and benefits of additional service consumption. Again returning to $\frac{\partial f_{11}^*}{\partial \eta_{11}} = -\frac{f_{11}^*}{\eta_{11}}(1 + \varepsilon_{s_1,\pi_1})$, we see that $-\frac{f_{11}^*}{\eta_{11}}\varepsilon_{s_1,\pi_1}$ represents the change in fuel usage attributable to the direct rebound effect. Therefore, the associated cost to society is this quantity times the marginal social cost: $-\frac{f_{11}^*}{\eta_{11}}\varepsilon_{s_1,\pi_1}(p + ke_1)$. Meanwhile, $\frac{\partial u}{\partial s_1}$ represents the benefits from additional service consumption, while $kc_1$ captures service externalities. The third cluster represents the analogous measure of net benefits (costs) accruing from indirect rebounds.

For intuition, we can fix ideas by letting $s_1$ be vehicle miles traveled (VMT) and $s_2$ be usage of household electronics. Pollution externalities arise from gasoline usage ($f_{11}$) and electricity usage ($f_{22}$), while congestion externalities arise from $s_1$ but not $s_2$ (so $c_2 = 0$). Recall from above that $\frac{\partial s_1^*}{\partial \eta_{11}} \geq 0$, so the congestion externality from driving necessarily increases. However, externalities from pollution may increase or decrease. Therefore the direct and indirect rebounds may improve or reduce welfare, and overall welfare may increase or decrease following an efficiency improvement. Since the welfare consequences are ambiguous a priori, it is worth further analysis. We begin by examining when rebounds are beneficial (or harmful) in and of themselves in response to the calls for rebound mitigation policies. Then, we will use this understanding to gain insight into the overall welfare consequences of an efficiency improvement.

## 5.1 Welfare effects of rebounds

As identified in (6), the net benefit of the direct rebound is

$$\frac{\varepsilon_{s_1,\pi_1}}{\eta_{11}}\left(f_{11}^*\left(p_1+ke_1\right)+s_1^*\left(kc_1-\frac{\partial u}{\partial s_1}\right)\right).$$

Substituting $\pi_1 s_1^*$ for $p_1 f_{11}^*$ and rearranging terms, we come to an equivalent expression:

$$-\frac{\varepsilon_{s_1,\pi_1}}{\eta_{11}}\left(\gamma_1 s_1^*-ke_1 f_{11}^*-kc_1 s_1^*\right), \tag{7}$$

where we define $\gamma_1 \equiv \frac{\partial u}{\partial s_1}-\pi_1$.

**Proposition 6.** *The direct rebound effect may increase or decrease overall welfare.*

*Proof.* Because $\varepsilon_{s_1,\pi_1}<0$, (7) is positive (negative) if the terms inside the brackets sum to be positive (negative). As such, the direct rebound effect is beneficial (detrimental) if $\gamma_1 > (<)\, \frac{ke_1}{\eta_{11}}+kc_1$. $\square$

We can conceptualize $\gamma_1$ as surplus from additional driving.[10] From this, the interpretation of our proposition becomes clear: the direct rebound is beneficial overall if the surplus from additional driving outweighs the external congestion and pollution costs that arise from that driving. The welfare impact of the indirect rebound can be analyzed in a similar way, but in this case the calculation is even simpler. Because $c_2=0$, we simply check whether $\gamma_2>\frac{ke_2}{\eta_{22}}$ in order to determine whether the indirect rebound is beneficial overall. For example, does the surplus gained from additional kWh of household electricity usage outweigh the externalities from upstream coal combustion?

Notably, the welfare implications of the direct rebound effect can be positive or negative under reasonable real-world estimates. While quantifying environmental externalities is

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[10] To be fully precise, $\frac{\partial u}{\partial s_i}=\pi_i$ at the margin according to the utility maximization problem; therefore, $\gamma_1=0$ at the margin. However, when we consider a discrete change from $\eta_{11}$ to $\eta_{11}'$, as would occur in a realistic policy scenario, consumer surplus will accrue along the change in $\eta_{11}$, as $\frac{\partial u}{\partial s_1}>\pi_1$ away from the margin. Thus, we will assume $\frac{\partial u}{\partial s_1}>\pi_1$ for simplicity of exposition, recognizing that it is not true on the margin. In this context, we can interpret $\frac{\partial u}{\partial s_1}$ as the average surplus change with the efficiency change.

notoriously difficult, Parry and Small (2005) suggest low-end estimates of external costs of driving of $ke_1 = 0.4$ cents/mile from pollution and $kc_1 = 1.5$ cents/mile from congestion.[11] Therefore, if the average surplus from additional driving is 5 cents/mile, the benefits from the direct rebound would outweigh the costs. However, using the high-end estimates of external costs for pollution (5.4 cents/mile) and congestion (15 cents/mile), the result is reversed. Thus, understanding the consumer value of driving is critical to understanding the welfare effects of the rebounds.

Based on values from this literature, the net effect of the indirect rebound will also be ambiguous and is context-specific. For example, estimated marginal pollution costs for coal-based electricity can range from 0.1 to 27 cents/kWh (Krupnick and Burtraw 1996; Epstein et al. 2011). Based on typical electricity prices, the benefits of electricity consumption on the margin will likely also fall in this range.

## 5.2 Overall welfare implications of energy efficiency improvements

How does the microeconomic rebound effect influence the welfare implications of an energy efficiency improvement? This is a question that has not been addressed in literature. Yet, arguably, this is a fundamental question for assessing the merits of an energy efficiency policy. A full analysis of energy efficiency policies would examine the cost of the policy and the net benefits of the policy including the rebound (as shown in (6)). In addition, there is much work in environmental economics about the "energy efficiency gap" and the possibility of behavioral failures that lead to underinvestment in energy efficiency and should be included in the welfare calculations (Gillingham and Palmer 2014). Such behavioral failures are outside the scope of this paper, but may also influence the social welfare implications of an energy efficiency policy. Combined, these effects can determine whether an energy efficiency policy is an inferior second-best policy or may even be preferred over price-based policies.

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[11] The marginal externality cost of congestion quoted here is interpreted as $kc_1$ rather than just $c_1$ in order to maintain consistency with the representative consumer framework that we have presented. The same interpretation is also used for other marginal externality cost values from the literature.

Our formal exposition of the welfare effects in (6) provides some guidance on how the rebound effect influences the welfare implications from improved energy efficiency. Some results are straightforward. When service externalities are large (e.g., in crowded transportation networks or in urban centers with significant noise pollution), welfare is more likely to decrease with an improvement in energy efficiency. When the direct rebound effect is large and the external costs of pollution are large, the energy efficiency improvement is more likely to decrease welfare. On the other hand, when the direct rebound effect is small or the external costs of pollution are small, the energy efficiency improvement is more likely to increase welfare.

Less obvious insights also emerge from (6). When indirect pollution costs $e_2$ are high (e.g., if dirty fuels are used to supply electricity), welfare is likely to increase (decrease) if $s_2$ is a gross substitute (complement) for $s_1$. In cases of backfire, welfare can only increase if the consumer surplus from the energy service consumption is particularly valuable.

Furthermore, it is useful to note that an energy efficiency improvement may improve welfare even if both the direct and indirect rebound effects reduce welfare. There is still the first term in (6) and there may also be behavioral failures (not explicitly included in our analysis). This underscores the point that an energy efficiency policy should not be dismissed simply because it results in a large rebound effect (or one with high external costs).

Finally, this exposition points to the likely importance of service externalities in the analysis of energy efficiency improvements and policies. Nearly all of the discussion of the impact of the rebound effect focuses entirely on implications for pollution and net energy use. From a welfare perspective, this is much too narrow. Consider an illustrative example where the representative consumer owns a vehicle with a fuel economy of 25 miles per gallon and drives 10,000 miles in a year. Suppose that congestion costs are low, at 3 cents per mile, and pollution externality costs are high at 5 cents per mile (for a 25 mpg car). Assume that the long-run elasticity of driving with respect to the implicit price happens to be $\varepsilon_{s_1,\pi_1} = -0.5$. Then a policy that doubles the fuel economy of the consumer's vehicle will cause the consumer to drive 5,000 additional miles, resulting in $150 in additional congestion costs. However, the pollution cost per mile is now 2.5 cents per mile, meaning that overall pollution costs have decreased from $500 to $375, for a net benefit of $125. An environmentally-minded policy-maker might support such this efficiency improvement as a means for ameliorating pollution. However, from a full welfare perspective, the congestion costs tip the balance, implying that this efficiency improvement is welfare-reducing.

# 6 Conclusions and Implications for Policy

This paper develops and analyzes a model of consumer energy usage decisions to provide new insight into the microeconomics of the rebound effect and its welfare implications. We present a clean framework and show how many of the findings in the literature rely on a special case in this framework: a one-to-one correspondence between fuels and energy services.

A noteworthy theme that emerges is the importance of complement and substitute relationships between energy services. These dictate the extent of the indirect rebound effect and mediate the potentially countervailing relationship between the direct and indirect rebound effects. Complement and substitute relationships for energy services turn out to be particularly meaningful when we have multiple fuels available for a single energy service or multiple energy services that use a single fuel. In these cases, the standard relationships between fuel price elasticities and the rebound effect no longer hold, and common methods of estimating the rebound effect will be biased, suggesting that researchers estimating the rebound effect should focus more on energy service price elasticities (e.g., the elasticity of VMT with respect to the price per mile of driving). Our theory identifies the cross-price term that governs the bias, highlighting that the commonly used elasticities based on fuel price variation are more likely to be valid in contexts with few substitutes or complements (e.g., personal vehicle travel in the United States). Uncovering these terms may also allow empirical economists to estimate bounds on the rebound effect based on fuel price variation.

This paper is the first to rigorously address externalities and social welfare in the context of the rebound effect. To date, there has been no analytical work on the merits of mitigating the rebound effect, even though the concept has been discussed extensively. Our analysis makes it possible to evaluate such efforts in a structured way, and it makes clear that rebound mitigation is not necessarily desirable, because rebounds can be beneficial or detrimental, depending on the balance between consumption benefits and external costs. For example, in developing countries or poor communities, where access to cheap energy services can be highly beneficial, efforts to constrain rebounds may in fact be quite harmful to social welfare. Although such measures may alleviate some of the environmental burdens arising from increased energy usage, they may do so at a social cost by denying access to valuable energy services.

From a broader welfare perspective, we show that energy efficiency improvements result in changes in consumption patterns that can be both welfare-reducing and welfare-improving. Our framework lays out the tradeoffs between consumption benefits, energy service externalities, and fuel externalities, while showing how each is related to the rebound effect. By enumerating the various welfare considerations in a careful way, we help guide clear thinking on the merits of energy efficiency improvements.

Our findings have important implications for policy. Consider fuel economy standards, which are a common policy context for discussing the rebound effect. Such a policy would be more effective at reducing energy use and emissions if driving demand is inelastic so the direct rebound effect is small and other energy services are substitutes for driving. However, if we broaden the policy criteria from energy usage to a more holistic consideration of social welfare, then we must also consider the change in benefits consumers receive from the policy, the incremental investment cost, and the change in external costs. The rebound effect actually *increases* social welfare, unless there are sufficiently large external costs from the additional fuel use or energy service use. This implies that before considering ancillary policy efforts to “mitigate” the rebound effect, we should first consider policies to address these external costs. If these are not possible, we should at least attempt to quantify these external costs and compare them to the consumer surplus gains from the additional energy service use.

This work provides a unified model and clear insights on the microeconomics of the rebound effect, while also suggesting several pathways for future research. Despite the demonstrated importance of complement and substitute relationships between energy services, there is extremely limited empirical evidence on such relationships. We could imagine randomized controlled trials to examine such relationships and help us pin down the indirect rebound effect in a more rigorous way. Furthermore, we have yet to see empirical research estimating the net welfare change from the rebound effect that weighs the consumer surplus gains against the external costs. We conclude by noting that this is a microeconomic treatment of the rebound effect. Economic logic suggests that similar complement and substitute relationships are also critical for determining the magnitude of the macroeconomic rebound effect, yet we have not seen any research exploring this path.

# Appendix

This appendix derives the welfare effects of an exogenous improvement in energy efficiency. Starting with

$$\frac{\partial SW}{\partial \eta_{11}} = -p_1 \frac{\partial f_{11}^*}{\partial \eta_{11}} - p_2 \frac{\partial f_{22}^*}{\partial \eta_{11}} + \frac{\partial u}{\partial s_1} \frac{\partial s_1^*}{\partial \eta_{11}} + \frac{\partial u}{\partial s_2} \frac{\partial s_2^*}{\partial \eta_{11}} - k \left( e_1 \frac{\partial f_{11}^*}{\partial \eta_{11}} + e_2 \frac{\partial f_{22}^*}{\partial \eta_{11}} + c_1 \frac{\partial s_1^*}{\partial \eta_{11}} + c_2 \frac{\partial s_2^*}{\partial \eta_{11}} \right),$$

we can use the following identities:

$$\begin{aligned}
\frac{\partial s_1^*}{\partial \eta_{11}} &= -\frac{s_1^*}{\eta_{11}} \varepsilon_{s_1,\pi_1} \\
\frac{\partial s_2^*}{\partial \eta_{11}} &= -\frac{s_2^*}{\eta_{11}} \varepsilon_{s_2,\pi_1} \\
\frac{\partial f_{11}^*}{\partial \eta_{11}} &= \frac{s_1^*}{\eta_{11}^2} \varepsilon_{f_{11},\eta_{11}} = -\frac{s_1^*}{\eta_{11}^2} (1 + \varepsilon_{s_1,\pi_1}) \\
\frac{\partial f_{22}^*}{\partial \eta_{11}} &= -\frac{s_2^*}{\eta_{11}\eta_{22}} \varepsilon_{s_2,\pi_1}.
\end{aligned}$$

Substituting elasticity expressions for derivatives, then substituting $f_{ii}^* = \frac{s_i^*}{\eta_{ii}}$, we obtain:

$$\begin{aligned}
\frac{\partial SW}{\partial \eta_{11}} &= \frac{p_1 s_1^*}{\eta_{11}^2}(1 + \varepsilon_{s_1,\pi_1}) + \frac{p_2 s_2^*}{\eta_{11}\eta_{22}} \varepsilon_{s_2,\pi_1} - \frac{\partial u}{\partial s_1^*} \frac{s_1}{\eta_{11}} \varepsilon_{s_1,\pi_1} - \frac{\partial u}{\partial s_2} \frac{s_2^*}{\eta_{11}} \varepsilon_{s_2,\pi_1} + \\
&\quad k \left( \frac{e_1 s_1^*}{\eta_{11}^2}(1 + \varepsilon_{s_1,\pi_1}) + \frac{e_2 s_2^*}{\eta_{11}\eta_{22}} \varepsilon_{s_2,\pi_1} + \frac{c_1 s_1^*}{\eta_{11}} \varepsilon_{s_1,\pi_1} + \frac{c_2 s_2^*}{\eta_{11}} \varepsilon_{s_2,\pi_1} \right) \\
\frac{\partial SW}{\partial \eta_{11}} &= \underbrace{\frac{f_{11}^* \left( p_1 + k e_1 \right)}{\eta_{11}} (1 + \varepsilon_{s_1,\pi_1})}_{\text{Due to change in fuel usage}} + \underbrace{\frac{\varepsilon_{s_1,\pi_1}}{\eta_{11}} \left( s_1^* \left( k c_1 - \frac{\partial u}{\partial s_1} \right) \right)}_{\text{Due to change in service usage}} + \qquad (8) \\
&\quad \underbrace{\frac{\varepsilon_{s_2,\pi_1}}{\eta_{11}} \left( f_{22}^* \left( p_2 + k e_2 \right) + s_2^* \left( k c_2 - \frac{\partial u}{\partial s_2} \right) \right)}_{\text{Due to indirect rebound}}
\end{aligned}$$

This expression explicitly captures the marginal welfare change in response to an exogenous improvement in energy efficiency. By examining the terms that comprise this expression, we can gain further insight on the benefits of rebounds as well as the tradeoffs between consumption and externalities.

The first cluster of terms denotes the change in societal costs attributable to a change in the use of $f_{11}^*$. If there an increase in fuel usage (i.e., direct rebound backfire where $\varepsilon_{s_1,\pi_1} < -1$), then this will cause utility to decrease. For rebounds short of backfire, usage of $f_{11}^*$ will decrease, leading to pollution reduction benefits as well as monetary savings.

The second cluster of terms denotes the change in societal costs and benefits due to a change in service 1 consumption. Given that $\varepsilon_{s_1,\pi_1} < 1$, service 1 consumption will increases, leading to losses from additional congestion but gains from consumption. Of course, the first two clusters are related to one another, as evidenced by the fact that both contain $\varepsilon_{s_1,\pi_1}$.

Whereas the first two clusters capture changes in the first fuel/service, the third cluster captures changes in the other fuel/service, i.e., the indirect rebound. The indirect rebound $\varepsilon_{s_2,\pi_1}$ can be positive or negative, depending on complementarity/substitutability. Moreover, the term in brackets can be positive or negative, depending on how consumption benefits $\frac{\partial u}{\partial s_2}$ compare with externality costs. Therefore, the indirect rebound could benefit or harm society overall.

By rearranging the terms between the first two clusters, we can also isolate the effects of the direct rebound effect to gain further insight, as described in the text:

$$\begin{aligned} \frac{\partial SW}{\partial \eta_{11}} = \quad & \underbrace{\frac{f_{11}^*}{\eta_{11}}(p_1 + ke_1)}_{\substack{\text{Averted social} \\ \text{costs with no rebound}}} \quad + \underbrace{\frac{\varepsilon_{s_1,\pi_1}}{\eta_{11}}\left(f_{11}^*\left(p_1 + ke_1\right) + s_1\left(kc_1 - \frac{\partial u}{\partial s_1^*}\right)\right)}_{\substack{\text{Net Benefits (Costs)} \\ \text{of Direct Rebound}}} \\ & + \underbrace{\frac{\varepsilon_{s_2,\pi_1}}{\eta_{11}}\left(f_{22}^*\left(p_2 + ke_2\right) + s_2^*\left(kc_2 - \frac{\partial u}{\partial s_2}\right)\right)}_{\substack{\text{Net Benefits (Costs)} \\ \text{of Indirect Rebound}}}. \qquad (9) \end{aligned}$$

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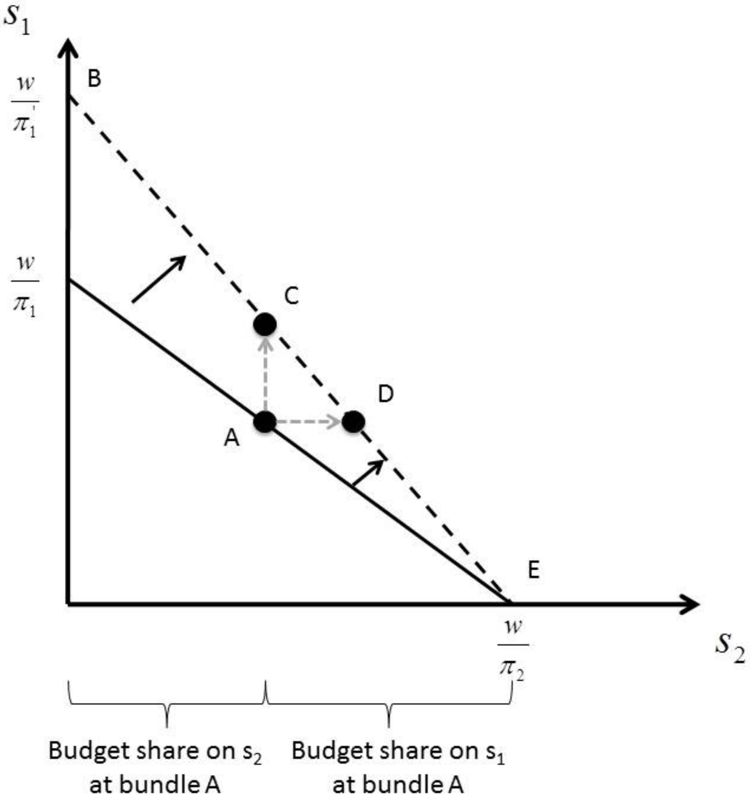

Figure 1: This figure illustrates how an efficiency change affects the budget constraint faced by consumers. The initial consumption bundle is A, on the initial budget constraint (solid line), and the energy efficiency improvement shifts the budget constraint to the dashed line. Under standard assumptions, the new consumption bundle will lie between B and D. If the new consumption bundle lies between C and B, then there is backfire.