Package 'capn'

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Description (Testing Version: Do not distribute!) A collection of functions that implements the approximation methods for natural captial asset prices suggested by Fenichel and Abbott (2014) in Journal of the Associations of Environmental and Resource Economists, Fenichel et al. (2016) in Proceedings of the National Academy of Sciences, and a third method, and its extensions to multiple stocks (where feasible): creating Chebyshev polynomial nodes and grids, calculating basis of Chebyshev polynomials, approximation and their simulations for: V-approximation (single and multiple stocks), P-approximation (single stock, PNAS), and Pdot-approximation (single stock, JAERE).

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Depends R (>= 3.0.1)

License GPL (>= 2)

LazyData true

RoxygenNote 5.0.1

NeedsCompilation no

R topics documented:

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aproxdef *Defining Approximation Space*

Description

The function defines an approximation space for all three approximation apporoaches (V, P, and Pdot).

Usage

```
aproxdef(deg, lb, ub, delta)
```

Arguments

deg	An array of degrees of approximation function: degrees of Chebyshev polynomials
lb	An array of lower bounds
ub	An array of upper bounds
delta	discount rate

Details

For the i-th dimension of $i = 1, 2, \cdots, d$, suppose a polynomial approximant s_i over a bounded interval $[a_i, b_i]$ is defined by Chebysev nodes. Then, a d-dimensional Chebyshev grids can be defined as:

$$\mathbf{S} = \{(s_1, s_2, \cdots, s_d) | a_i \leq s_1 \leq b_i, i = 1, 2, \cdots, d\}.$$

Suppose we impletement n_i numbers of polynomials (i.e., $(n_i - 1)$-th order) for the i-th dimension. The approximation space is defined as:

`deg = c(`$n_1, n_2, \cdots, n_d$`)`,
`lb = c(`$a_1, a_2, \cdots, a_d$`)`, and
`ub = c(`$b_1, b_2, \cdots, b_d$`)`.

`delta` is the given constant discount rate.

Value

A list of approximation space

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "Natural Capital: From Metaphor to Measurement." *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

See Also

vaprox, vsim, paprox, psim, pdotaprox, pdotsim

Examples

```
## Reef-fish example: see Fenichel and Abbott (2014)
delta <- 0.02
upper <- 359016000     # upper bound on approximation space
lower <- 5*10^6        # lower bound on approximation space
myspace <- aproxdef(50,lower,upper,delta)
## Two dimensional example
ub <- c(1.5,1.5)
lb <- c(0.1,0.1)
deg <- c(20,20)
delta <- 0.03
myspace <- aproxdef(deg,lb,ub,delta)
```

---

chebbasisgen — *Generating Unidimensional Chebyshev polynomial (monomial) basis*

Description

The function calculates the monomial basis of Chebyshev polynomials for the given unidimensional nodes, s_i, over a bounded interval [a,b].

Usage

```
chebbasisgen(stock, npol, a, b, dorder = NULL)
```

Arguments

`stock`	An array of Chebyshev polynomial nodes s_i (an array of stocks in capn-packages)
`npol`	Number of polynomials (n polynomials = (n-1)-th degree)
`a`	The lower bound of inverval [a,b]
`b`	The upper bound of inverval [a,b]
`dorder`	Degree of partial derivative of the basis; Default is NULL; if dorder = 1, returns the first order partial derivative

Details

Suppose there are m numbers of Chebyshev nodes over a bounded interval [a,b]:

$s_i \in [a,b]$, for $i = 1, 2, \cdots, m$.

These nodes can be nomralized to the standard Chebyshev nodes over the domain [-1,1]:

$z_i = \frac{2(s_i - a)}{(b-a)} - 1$.

With normalized Chebyshev nodes, the recurrence relations of Chebyshev polynomials of other n is defined as:

$T_0(z_i) = 1$,
$T_1(z_i) = z_i$, and
$T_n(z_i) = 2z_iT_{n-1}(z_i) - T_{n-2}(z_i)$.

The interpolation matrix (Vandermonde matrix) of (n-1)-th Chebyshev polynomials with m numbers of nodes, Φ_{mn} is:

$$\Phi_{mn} = \begin{bmatrix} 1 & T_1(z_1) & \cdots & T_{n-1}(z_1) \\ 1 & T_1(z_2) & \cdots & T_{n-1}(z_2) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & T_1(z_m) & \cdots & T_{n-1}(z_m) \end{bmatrix}.$$

The partial derivative of the monomial basis matrix can be found by the relation:

$(1 - z_i^2)T_n'(z_i) = n[T_{n-1}(z_i) - z_iT_n(z_i)]$.

The technical details of the monomial basis of Chebyshev polynomial can be referred from Amparo et al. (2007) and Miranda and Fackler (2012).

Value

A matrix (number of nodes (m) x npol (n)) of (monomial) Chebyshev polynomial basis

References

Amparo, Gil, Javier Segura, and Nico Temme. (2007) *Numerical Methods for Special Functions*. Cambridge: Cambridge University Press.
Miranda, Mario J. and Paul L. Fackler. (2002) *Applied Computational Economics and Finance*. Cambridge: The MIT Press.

See Also

chebnodegen

Examples

```
## Reef-fish example: see Fenichel and Abbott (2014)
data("reefdata")
stock <- reefaproxdata[1:10,1]
## An example of Chebyshev polynomial basis
chebbasisgen(stock,20,0.1,1.5)
## The partial derivative of Chebyshev polynomial basis with the same function
chebbasisgen(stock,20,0.1,1.5,1)
```

`chebgrids` *Generating Chebyshev grids*

Description

This function generates a grid of multi-dimensional Chebyshev nodes.

Usage

```
chebgrids(nnodes, lb, ub, rtype = NULL)
```

Arguments

nnodes	An array of numbers of nodes
lb	An array of lower bounds
ub	An array of upper bounds
rtype	A type of results; default is NULL that returns a list class; if rtype = list, returns a list class; if rtype = grid, returns a matrix class.

Details

For the i-th dimension of $i = 1, 2, \cdots, d$, suppose a polynomial approximant s_i over a bounded interval $[a_i, b_i]$ is defined by Chebysev nodes. Then, a d-dimensional Chebyshev grids can be defined as:

$$\mathbf{S} = \{(s_1, s_2, \cdots, s_d) | a_i \leq s_1 \leq b_i, i = 1, 2, \cdots, d\}.$$

This is all combinations of s_i. Two types of results are provided. 'rtype = list' provides a list of d dimensions wherease 'rtype = grids' creates a $\left(\prod_{i=1}^{d} n_i\right) \times d$ matrix.

Value

A list with d elements of Chebyshev nodes or a $\left(\prod_{i=1}^{d} n_i\right) \times d$ matrix of Chebyshev grids

See Also

chebnodegen

Examples

```
## Chebyshev grids with two-dimension
chebgrids(c(5,3), c(1,1), c(2,3))
# Returns the same results
chebgrids(c(5,3), c(1,1), c(2,3), rtype='list')
## Returns a matrix grids with the same domain
chebgrids(c(5,3), c(1,1), c(2,3), rtype='grid')
## Chebyshev grids with one-dimension
chebgrids(5,1,2)
chebnodegen(5,1,2)
## Chebyshev grids with three stock
chebgrids(c(3,4,5),c(1,1,1),c(2,3,4),rtype='grid')
```

chebnodegen — *Unidimensional Chebyshev nodes*

Description

The function generates uni-dimensional chebyshev nodes.

Usage

```
chebnodegen(n, a, b)
```

Arguments

n	A number of nodes
a	The lower bound of inverval [a,b]
b	The upper bound of interval [a,b]

Details

A polynomial approximant s_i over a bounded interval [a,b] is constructed by:

$s_i = \frac{b+a}{2} + \frac{b-a}{2} cos(\frac{n-i+0.5}{n}\pi)$ for $i = 1, 2, \cdots, n$.

More detail explanation can be refered from Miranda and Fackler (2002, p.119).

Value

An array n Chebyshev nodes

References

Miranda, Mario J. and Paul L. Fackler. (2002) *Applied Computational Economics and Finance*. Cambridge: The MIT Press.

Examples

```
## 10 Chebyshev nodes in [-1,1]
chebnodegen(10,-1,1)
## 5 Chebyshev nodes in [1,5]
chebnodegen(5,1,5)
```

`lvdata` *Prey-Predator (Lotka-Volterra) example: two stocks (forthcoming)*

Description

(Note: The data will be available in the next version!) The `lvdata` provides the data to simulate prey-predator (Lotka-Volterra) model. The original code was written by Josha Abbott in MATLAB. Seong Do Yun adapted it to a package example. This dataset consists of two data.frames: lvaproxdata and lvsimdata_time.

Usage

```
data("lvdata")
```

Format

lvaproxdata: a data.frame for approximation (evaluated on (20 x 20) Chebyshev nodes)

- `xs` prey stock
- `xy` predator stock
- `xdot` evaluated xdot $\frac{dx}{dt}$
- `ydot` evaluated ydot $\frac{dy}{dt}$
- `wval` profit (W in Fenichel and Abtott (2014))

lvsimdata.time: a data for time simulation (101 ODE solution)

- `tseq` time sequence from 0 to 100
- `xs` prey stock
- `ys` predator stock

`paprox` *Calculating P-approximation coefficients*

Description

The function provides the P-approximation coefficients of the defined Chebyshev polynomials in `aproxdef`. For now, only unidimensional case is developed.

Usage

```
paprox(stock, sdot, dsdotds, dwds, aproxspace)
```

Arguments

`stock`	An array of stock, s
`sdot`	An array of ds/dt, $\dot{s} = \frac{ds}{dt}$
`dsdotds`	An array of d(sdot)/ds, $\frac{d\dot{s}}{ds}$
`dwds`	An array of dw/ds, $\frac{dW}{ds}$
`aproxspace`	An approximation space defined by `aproxdef` function

Details

The P-approximation is finding the shadow price of a stock, p from the relation:

$p(s) = \frac{W_s(s)+\dot{p}(s)}{\delta - \dot{s}_s}$,

where $W_s = \frac{dW}{ds}$, $\dot{p}(s) = \frac{dp}{ds}$, $\dot{s}_s = \frac{d\dot{s}}{ds}$, and δ is the given discount rate.

Consider approximation $p(s) = \mu(s)\beta$, $\mu(s)$ is Chebyshev polynomials and β is their coeffcients. Then, $\dot{p} = \mu_s(s)\beta$ by the orthogonality of Chebyshev basis. Adopting the properties above, we can get the unknown coefficient vector β from:

$\mu\beta = (W_s + \dot{s}\mu_s\beta)(\delta - \dot{s}_s)^{-1}$, and thus,

$\beta = (\delta\mu - \dot{s}_s\mu - \dot{s}\mu_s)^{-1} W_s$.

#' In a case of over-determined (more nodes than approaximation degrees),

$\left((\delta\mu - \dot{s}_s\mu - \dot{s}\mu_s)^T (\delta\mu - \dot{s}_s\mu - \dot{s}\mu_s)\right)^{-1} (\delta\mu - \dot{s}_s\mu - \dot{s}\mu_s)^T W_s$

For more detils see Fenichel and Abbott (2014) and Fenichel et al. (2016).

Value

A list of approximation resuts: deg, lb, ub, delta, and coefficients

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "Natural Capital: From Metaphor to Measurement." *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

See Also

aproxdef, psim

Examples

```
## 1-D Reef-fish example: see Fenichel and Abbott (2014)
data("reefdata")
st <- reefaproxdata[,1]
sdot <- reefaproxdata[,2]
dsdotds <- reefaproxdata[,3]
dwds <- reefaproxdata[,6]
aproxdeg <- 50
lower <- 5*10^6
upper <- 359016000
delta <- 0.02
reefspace <- aproxdef(aproxdeg,lower,upper,delta)
paproxc <- paprox(st,sdot,dsdotds,dwds,reefspace)
```

`pdotaprox` *Calculating Pdot-approximation coefficients*

Description

The function provides the Pdot-approximation coefficients of the defined Chebyshev polynomials in `aproxdef`. For now, only unidimensional case is developed.

Usage

```
pdotaprox(stock, sdot, dsdotds, dsdotdss, dwds, dwdss, aproxspace)
```

Arguments

`stock`	An array of stock, s
`sdot`	An array of ds/dt, $\dot{s} = \frac{ds}{dt}$
`dsdotds`	An array of d(sdot)/ds, $\frac{d\dot{s}}{ds}$
`dsdotdss`	An array of d/ds(d(sdot)/ds), $\frac{d}{ds}\left(\frac{d\dot{s}}{ds}\right)$
`dwds`	An array of dw/ds, $\frac{dW}{ds}$
`dwdss`	An array of d/ds(dw/ds), $\frac{d}{ds}\left(\frac{dW}{ds}\right)$
`aproxspace`	An approximation space defined by `aproxdef` function

Details

The Pdot-approximation is finding the shadow price of a stock, p from the relation:

$$p(s) = \frac{W_s(s)+\dot{p}(s)}{\delta-\dot{s}_s},$$

where $W_s = \frac{dW}{ds}$, $\dot{p}(s) = \frac{dp}{ds}$, $\dot{s}_s = \frac{d\dot{s}}{ds}$, and δ is the given discount rate.

In order to operationhalize this approach, we take the time derivative of this expression:

$$\dot{p} = \frac{((W_{ss}\dot{s}+\ddot{p})(\delta-\dot{s}_s)+(W_s+\dot{p})(\dot{s}_{ss}\dot{s}))}{(\delta-\dot{s}_s)^2}$$

Consider approximation $\dot{p}(s) = \mu(s)\beta$, $\mu(s)$ is Chebyshev polynomials and β is their coeffcients. Then, $\ddot{p} = \frac{d\dot{p}}{ds}\frac{ds}{dt} = \dot{s}\mu_s(s)\beta$ by the orthogonality of Chebyshev basis. Adopting the properties above, we can get the unknown coefficient vector β from:

$$\mu\beta = (\delta - \dot{s}_s)^{-2}\left[(W_{ss}\dot{s} + \dot{s}\mu_s\beta)(\delta - \dot{s}_s) + (W_s + \mu\beta)(\dot{s}_{ss}\dot{s})\right], \text{ and}$$

$$\beta = \left[(\delta - \dot{s}_s)^2\mu - \dot{s}(\delta - \dot{s}_s)\mu_s - \dot{s}_{ss}\dot{s}\mu\right]^{-1}(W_{ss}\dot{s}(\delta - \dot{s}_s) + W_s\dot{s}_{ss}\dot{s}).$$

If we suppose $A = \left[(\delta - \dot{s}_s)^2\mu - \dot{s}(\delta - \dot{s}_s)\mu_s - \dot{s}_{ss}\dot{s}\mu\right]$ and $B = (W_{ss}\dot{s}(\delta - \dot{s}_s) + W_s\dot{s}_{ss}\dot{s})$, then over-determined case can be calculated:

$$\beta = \left(A^TA\right)^{-1}A^TB.$$

For more detils see Fenichel and Abbott (2014) and Fenichel et al. (2016).

Value

A list of approximation results: deg, lb, ub, delta, and coefficients

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "Natural Capital: From Metaphor to Measurement." *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

See Also

`aproxdef`, `pdotsim`

Examples

```
## 1-D Reef-fish example: see Fenichel and Abbott (2014)
data("reefdata")
st <- reefaproxdata[,1]
sdot <- reefaproxdata[,2]
dsdotds <- reefaproxdata[,3]
dsdotdss <- reefaproxdata[,4]
dwds <- reefaproxdata[,6]
dwdss <- reefaproxdata[,7]
aproxdeg <- 50
lower <- 5*10^6
upper <- 359016000
delta <- 0.02
reefspace <- aproxdef(aproxdeg,lower,upper,delta)
pdotaproxc <- pdotaprox(st,sdot,dsdotds, dsdotdss,dwds, dwdss,reefspace)
```

---

pdotsim *Simulation of Pdot-approximation*

Description

The function provides the Pdot-approximation simulation.

Usage

```
pdotsim(stock, sdot, dsdotds, wval, dwds, pdotcoeff)
```

Arguments

`stock`	An array of stock
`sdot`	An array of ds/dt, $\dot{s} = \frac{ds}{dt}$
`dsdotds`	An array of d(sdot)/ds, $\frac{d\dot{s}}{ds}$
`wval`	An array of W-value
`dwds`	An array of dw/ds, $\frac{dW}{ds}$
`pdotcoeff`	An approximation result from pdotaprox function

Details

Let $\hat{\beta}$ be the approximation coefficent from the results of pdotaprox function. The estimated shadow price (accounting) price of stock over the given approximation intervals of $s \in [a, b]$, $\hat{p}$ can be calculated as:

$\hat{p} = \frac{W_s + \mu\beta}{\delta - \dot{s}_s}$.

The estimated inclusive wealth is:

$IW = \hat{p}s$.

The estimated value function is:

$\hat{V} = \frac{1}{\delta}(W + \hat{p}\dot{s})$.

For more detils see Fenichel and Abbott (2014) and Fenichel et al. (2016).

Value

A list of approximation resuts: shadow (accounting) prices, inclusive wealth, and Value function

See Also

pdotaprox

Examples

```
## 1-D Reef-fish example: see Fenichel and Abbott (2014)
data("reefdata")
st <- reefaproxdata[,1]
sdot <- reefaproxdata[,2]
dsdotds <- reefaproxdata[,3]
dsdotdss <- reefaproxdata[,4]
dwds <- reefaproxdata[,6]
dwdss <- reefaproxdata[,7]
aproxdeg <- 50
lower <- 5*10^6
upper <- 359016000
delta <- 0.02
reefspace <- aproxdef(aproxdeg,lower,upper,delta)
pdotaproxc <- pdotaprox(st,sdot,dsdotds, dsdotdss,dwds, dwdss,reefspace)

# approximation domain
st1 <- reefsimdata[,1]
sdot1 <- reefsimdata[,2]
dsdotds1 <- reefsimdata[,3]
wval1 <- reefsimdata[,5]
dwds1 <- reefsimdata[,6]
reefsim <- pdotsim(st1,sdot1,dsdotds1,wval1,dwds1,pdotaproxc)

# Accounting Price
plot(st1,reefsim[["shadowp"]],type='l', lwd=2, col="blue",
     ylim = c(0,30),
     xlab="Stock size, s",
     ylab="Accounting price")
```

```
# Inclusive Wealth
plot(st1,reefsim[["iw"]],type='l', lwd=2, col="blue",
     xlab="Stock size, s",
     ylab="Inclusive Wealth")
```

---

psim *Simulation of P-approximation*

Description

The function provides the P-approximation simulation.

Usage

```
psim(stock, pcoeff)
```

Arguments

stock	An array of stock variable
pcoeff	An approximation result from paprox function

Details

Let $\hat{\beta}$ be the approximation coefficent from the results of paprox function. The estimated shadow price (accounting) price of stock over the given approximation intervals of $s \in [a, b]$, $\hat{p}$ can be calculated as:

$\hat{p} = \mu(s)\hat{\beta}.$

The inclusive wealth is:

$IW = \hat{p}s.$

For more detils see Fenichel and Abbott (2014) and Fenichel et al. (2016).

Value

A list of approximation resuts: shadow (accounting) prices and inclusive wealth

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "Natural Capital: From Metaphor to Measurement." *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

See Also

aproxdef, paprox

Examples

```
## 1-D Reef-fish example: see Fenichel and Abbott (2014)
data("reefdata")
st <- reefaproxdata[,1]
sdot <- reefaproxdata[,2]
dsdotds <- reefaproxdata[,3]
dwds <- reefaproxdata[,6]
aproxdeg <- 50
lower <- 5*10^6
upper <- 359016000
delta <- 0.02
reefspace <- aproxdef(aproxdeg,lower,upper,delta)
paproxc <- paprox(st,sdot,dsdotds,dwds,reefspace)

# approximation domain
stapprox <- reefsimdata[,1]
reefsim <- psim(stapprox,paproxc)

# Accounting Price
plot(stapprox,reefsim[["shadowp"]],type='l', lwd=2, col="blue",
     ylim = c(0,30),
     xlab="Stock size, s",
     ylab="Accounting price")

# Inclusive Wealth
plot(stapprox,reefsim[["iw"]],type='l', lwd=2, col="blue",
     xlab="Stock size, s",
     ylab="Inclusive Wealth")
```

`reefdata` *Reef Fish example: one dimensional stock*

Description

The `reefdata` provides data to replicate the Gulf of Mexico Reef Fish example in Fenichel and Abbott (2014). This dataset is consisted of two data.frames: reefaproxdata and reefsimdata. Details will be updated including equations.

Usage

```
data("reefdata")
```

Format

reefaproxdata: a data.frame for approximation (evaluated on 50 Chebyshev nodes)

- `st` reef-fish stocks (generated from chebnodegen)
- `sdot` evaluated $\frac{dst}{dt}$
- `dsdotds` evaluated $\frac{dsdot}{ds}$
- `dsdotdss` evaluated $\frac{d}{ds}(\frac{dsdot}{ds})$
- `wval` profit (w in Fenichel and Abtott (2014))

- dwds evaluated $\frac{dw}{ds}$
- dwdss evaluated $\frac{d}{ds}(\frac{dw}{ds})$

reefsimdata: a data for simulation (502 evenly gridded values)

- st1 reef-fish stocks (generated from chebnodegen)
- sdot1 evaluated $\frac{dst1}{dt}$
- dsdotds1 evaluated $\frac{dsdot1}{ds}$
- dsdotdss1 evaluated $\frac{d}{ds}(\frac{dsdot1}{ds})$
- wval1 profit (W in Fenichel and Abtott (2014))
- dwds1 evaluated $\frac{dw}{ds}$
- dwdss1 evaluated $\frac{d}{ds}(\frac{dw}{ds})$

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "Natural Capital: From Metaphor to Measurement." *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

---

vaprox — *Calculating V-approximation coefficients*

Description

The function provides the V-approximation coefficients of the defined Chebyshev polynomials in aproxdef.

Usage

```
vaprox(sdata, aproxspace)
```

Arguments

sdata	A data.frame or matrix of [stock,sdot,benefit]=[$\mathbf{S}$,$\dot{\mathbf{S}}$,W]
aproxspace	An approximation space defined by aproxdef function

Details

The V-approximation is finding the shadow price of i-th stock, p_i for $i = 1, \cdots, d$ from the relation:

$$\delta V = W(\mathbf{S}) + p_1\dot{s}_1 + p_2\dot{s}_2 + \cdots + p_d\dot{s}_d,$$

where δ is the given discount rate, V is the value function, $\mathbf{S} = (s_1, s_2, \cdots, s_d)$ is a vector of stocks, $W(\mathbf{S})$ is the net benefits accruing to society, and $\dot{s}_i$ is the growth of stock s_i. By the definition of the shadow price, we know:

$$p_i = \frac{\partial V}{\partial s_i}.$$

Consider approximation $V(\mathbf{S}) = \mu(\mathbf{S})\beta$, $\mu(\mathbf{S})$ is Chebyshev polynomials and β is their coefficients. Then, $p_i = \mu_{s_i}(\mathbf{S})\beta$ by the orthogonality of Chebyshev basis. Adopting the properties above, we can get the unknown coefficient vector β from:

$$\delta\mu(\mathbf{S})\beta = W(\mathbf{S}) + \sum_{i=1}^{d} \mu_{s_i}(\mathbf{S})\beta, \text{ and thus,}$$

$$\beta = \left(\delta\mu(\mathbf{S}) - \sum_{i=1}^{d} \mu_{s_i}(\mathbf{S})\right)^{-1} W(\mathbf{S}).$$

In a case of over-determined (more nodes than approaximation degrees),

$$\beta = \left(\left(\delta\mu(\mathbf{S}) - \sum_{i=1}^{d} \mu_{s_i}(\mathbf{S})\right)^{T} \left(\delta\mu(\mathbf{S}) - \sum_{i=1}^{d} \mu_{s_i}(\mathbf{S})\right)\right)^{-1} \left(\delta\mu(\mathbf{S}) - \sum_{i=1}^{d} \mu_{s_i}(\mathbf{S})\right)^{T} W(\mathbf{S}).$$

For more detils see Fenichel and Abbott (2014) and Fenichel et al. (2016).

Value

A list of approximation resuts: deg, lb, ub, delta, and coefficients

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "Natural Capital: From Metaphor to Measurement." *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

See Also

aproxdef, vsim

Examples

```
## 1-D Reef-fish example: see Fenichel and Abbott (2014)
data("reefdata")
aproxdata <- cbind(reefaproxdata[,1],reefaproxdata[,2],reefaproxdata[,5])
aproxdeg <- 20
lower <- 5*10^6
upper <- 359016000
delta <- 0.02
reefspace <- aproxdef(aproxdeg,lower,upper,delta)
vaproxc <- vaprox(aproxdata,reefspace)

## 2-D Prey-Predator example
data("lvdata")
aproxdeg <- c(20,20)
lower <- c(0.1,0.1)
upper <- c(1.5,1.5)
delta <- 0.03
lvspace <- aproxdef(aproxdeg,lower,upper,delta)
vaproxc <- vaprox(lvaproxdata,lvspace)
```

vsim *Simulation of V-approximation*

Description

The function provides the V-approximation simulation by adopting the results of `vaprox`. Available for multiple stock problems.

Usage

```
vsim(adata, vcoeff)
```

Arguments

`adata`	A data.frame or matrix of [stock,sdot,benefit]=[$\mathbf{S}$,$\dot{\mathbf{S}}$,W]
`vcoeff`	An approximation result from `varpox` function

Details

Let $\hat{\beta}$ be the approximation coefficent from the results of `vaprox` function. The estimated shadow (accounting) price of i-th stock over the given approximation intervals of $s_i \in [a_i, b_i]$, $\hat{p}_i$ can be calcuated as:

$\hat{p}_i = \mu(\mathbf{S})\hat{\beta}$ where $\mu(\mathbf{S})$ Chebyshev polynomial basis.

The Inclusive wealth of i-th stock is:

$IW = \sum_i^d \hat{p}_i s_i$, and

the value function is:

$\hat{V} = \delta\mu(\mathbf{S})\hat{\beta}$.

For more detils see Fenichel and Abbott (2014) and Fenichel et al. (2016).

Value

A list of simulation resuts: shadow (accounting) prices, inclusive wealth, and Value function

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "Natural Capital: From Metaphor to Measurement." *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

See Also

`aproxdef`, `vsim`

Examples

```
## 1-D Reef-fish example: see Fenichel and Abbott (2014)
data("reefdata")
# Note vcol function requries a data.frame or matrix!
aproxdata <- matrix(cbind(reefaproxdata[,1],reefaproxdata[,2],reefaproxdata[,5]),ncol=3)
aproxdeg <- 20
lower <- 5*10^6
upper <- 359016000
delta <- 0.02
reefspace <- aproxdef(aproxdeg,lower,upper,delta)
vaproxc <- vaprox(aproxdata,reefspace)
stapprox <- matrix(reefsimdata[,1])
reefsim <- vsim(stapprox,vaproxc)

# plot shadow (accounting) price: Figure 4 in Fenichel and Abbott (2014)
plot(stapprox,reefsim[["shadowp"]],type='l', lwd=2, col="blue",
     ylim = c(0,15),
     xlab="Stock size, s",
     ylab="Accounting price")

## 2-D Prey-Predator example
data("lvdata")
aproxdeg <- c(20,20)
lower <- c(0.1,0.1)
upper <- c(1.5,1.5)
delta <- 0.03
lvspace <- aproxdef(aproxdeg,lower,upper,delta)
lvaproxc <- vaprox(lvaproxdata,lvspace)
lvsim <- vsim(lvsimdata.time[,2:3],lvaproxc)

# plot Biomass
plot(lvsimdata.time[,1], lvsimdata.time[,2], type='l', lwd=2, col="blue",
     xlab="Time",
     ylab="Biomass")
lines(lvsimdata.time[,1], lvsimdata.time[,3], lwd=2, col="red")
legend("topright", c("Prey", "Predator"), col=c("blue", "red"),
       lty=c(1,1), lwd=c(2,2), bty="n")

# plot shadow (accounting) prices
plot(lvsimdata.time[,1],lvsim[["shadowp"]][,1],type='l', lwd=2, col="blue",
     ylim = c(-5,7),
     xlab="Time",
     ylab="Accounting price")
lines(lvsimdata.time[,1],lvsim[["shadowp"]][,2], lwd=2, col="red")
legend("topright", c("Prey", "Predator"), col=c("blue", "red"),
       lty=c(1,1), lwd=c(2,2), bty="n")

# plot inclusive weath and value function
plot(lvsimdata.time[,1],lvsim[["iw"]],type='l', lwd=2, col="blue",
     ylim = c(-0.5,1.2),
     xlab="Time",
     ylab="Inclusive Wealth / Value Function ($)")
lines(lvsimdata.time[,1],lvsim[["vfun"]], lwd=2, col="red")
legend("topright", c("Inclusive Wealth", "Value Function"),
       col=c("blue", "red"), lty=c(1,1), lwd=c(2,2), bty="n")
```

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