Green Markets and Private Provision of Public Goods

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This paper develops a general model of private provision of a public good that includes the option to consume an impure public good. The model is used to investigate the positive and normative consequences of "green markets," which are based on technologies with joint production of a private good and an environmental public good. It is shown that under reasonable conditions green markets can have beneficial or detrimental effects on environmental quality and social welfare. The analysis applies equally to nonenvironmental choice settings, with examples ranging from socially responsible investments to commercial activities associated with charitable fund-raising.

I. Introduction

The economics literature on private provision of public goods has grown extensively over the last 25 years. The general assumption of theoretical research in this area is that individuals choose between consumption of a private good and contributions to a pure public good.[1] Models based on this assumption establish the foundation for understanding privately provided public goods. Yet individuals increasingly face a third option: consumption of impure public goods that generate private and public goods as a joint product. This paper addresses fundamental ques-

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[1] Standard treatments can be found in Bergstrom, Blume, and Varian (1986), Andreoni (1988), and Cornes and Sandler (1996).

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tions about how the option to consume impure public goods affects private provision and social welfare.

Markets for "environmentally friendly" goods and services exemplify the increased availability of impure public goods in the economy. The distinguishing feature of these markets—hereafter referred to as "green markets"—is availability of impure public goods (i.e., green goods) that arise through joint production of a private good and an environmental public good. For example, consider the growing market for "green electricity," which is electricity generated with renewable sources of energy. Consumers increasingly have the option to purchase green electricity with a price premium that applies to all or part of their household's electricity consumption. In return, production of green electricity displaces pollution emissions from electricity generated with fossil fuels. Thus consumers of green electricity purchase a joint product—electricity consumption and reduced emissions. Another example is the growing market for premium-priced, shade-grown coffee, which is coffee grown under the canopy of tropical forests rather than in open, deforested fields. Shade-grown coffee plantations provide important refuges for tropical biodiversity, including migratory birds. Thus consumers of shade-grown coffee also purchase a joint product—coffee consumption and biodiversity conservation.

More generally, green markets are expanding in many sectors of the economy in response to a willingness-to-pay premium for goods and services with environmental benefits. According to market research in the United States, green products account for 9.5 percent of all new-product introductions in the economy (Marketing Intelligence Service 1999), and analysts have identified the growth and opportunities in green markets as the "next big thing" for small business (Murphy 2003). Furthermore, expansion of green markets worldwide has prompted certification and labeling programs that cover thousands of products in dozens of countries. Contributing to these trends is the fact that many governments, nongovernmental organizations, and industries promote green markets as a decentralized mechanism to encourage private provision of environmental public goods.

Beyond green markets, it is increasingly common to see joint products with private and public characteristics of various types. In many cases firms simply donate a percentage of their profits to a charitable cause. This practice ranges from goods such as cosmetics and ice cream to services such as credit cards and long-distance telecommunication. Furthermore, many charitable and nonprofit organizations finance their activities, in part, through the sale of private goods, such as theater tickets and magazine subscriptions. Finally, opportunities for "socially responsible" investing combine a positive externality with an investment return. In all these examples, the joint product forms an impure public good—with private and public characteristics.

This paper develops a general model of private provision of a public good that includes the option to consume an impure public good. Building on the characteristics approach to consumer behavior (Lancaster 1971; Gorman 1980), the model assumes that individuals derive utility from characteristics of goods rather than from goods themselves. The choice setting is such that individuals have the opportunity to consume a private good and make a contribution to a pure public good, with each activity generating its own characteristic. Additionally, the same private and public characteristics are available jointly through consumption of an impure public good.

The distinguishing feature of the model is the way in which characteristics are available through more than one activity. As noted above, the standard pure public good model has only a private good and a pure public good. In the standard impure public good model, the characteristics of the impure public good are not available through any other means (Cornes and Sandler 1984, 1994). This setup has been extended in other models to enable provision of the public characteristic through direct donations (Vicary 1997, 2000), but the private characteristic of the impure public good remains otherwise unavailable. In contrast, the model developed here applies when both characteristics of the impure public good are also available separately.[2]

This generalization of the choice setting enables broad application of the model. In the context of green markets—the application referred to throughout the paper—the model captures the fact that individuals typically have three relevant choices: a conventional (pure private) good, a direct donation to an environmental (pure public) good, and a green (impure public) version of the good that jointly provides characteristics of the other two choices. For instance, consumers of green electricity have options to purchase conventional electricity and donate directly to reduce emissions. Similarly, consumers of shade-grown coffee have options to purchase conventional coffee and make donations to conserve tropical biodiversity.

After developing the model in Section II, the paper focuses on three primary questions: Will green markets actually lead to improvements in environmental quality? How does the potential for induced changes in environmental quality depend on the number of individuals in the economy? And how will green markets affect social welfare? While the model is motivated and discussed using green markets and environmental qual-

[2] In another paper (Kotchen 2005), I consider a similar setup, but the analysis focuses on the comparative statics of individual behavior. The present paper focuses on equilibrium results.

ity, readers should keep in mind that the results apply to any market setting that includes an impure public good with alternative ways to obtain its joint products.

Several of the results are quite striking. Despite the intent of green markets to improve environmental quality, it is shown that under reasonable conditions, introducing a green market or improving a green technology can actually discourage private provision of an environmental public good. If, however, the economy is sufficiently large in terms of the number of individuals or environmental quality is a gross complement for private consumption, this counterintuitive result is no longer possible. Another surprising implication of the model is that green markets can be welfare-immiserizing, despite the fact that they expand both the choice set over market goods and the production possibilities over characteristics. Overall, the results have implications for public policy related to the role of green markets as a mechanism to improve environmental quality. The findings also contribute more generally to the understanding of how impure public goods affect private provision and social welfare.

II. The Model

There are $i = 1, \dots, n$ individuals in the economy. Individuals derive utility from characteristics of goods rather than from the goods themselves. Assume for simplicity that there are two characteristics, X and Y. Characteristic X has properties of a composite private good, and characteristic Y has properties of a pure public good. We can interpret Y as environmental quality. Each individual's preferences are represented by a strictly increasing and strictly quasi-concave utility function $U_i = U_i(X_i, Y)$, where X_i is individual i's private consumption of X, and Y is aggregate provision of the public characteristic such that $Y = \sum_{i=1}^{n} Y_i$, where Y_i is individual i's private provision.[3]

Each individual is endowed with exogenous wealth $w_i > 0$, which can be allocated among three market goods: a conventional good c_i that generates X_i, a direct donation d_i that generates Y_i, and a green (or impure public) good g_i that generates X_i and Y_i jointly. To simplify notation, choose units of c_i, d_i, and g_i to normalize all prices to unity. Moreover, choose units of X_i and Y_i such that one unit of c_i generates one unit of X_i and one unit of d_i generates one unit of Y_i. Let $\alpha > 0$ and $\beta > 0$ characterize the green technology such that one unit of g_i generates α units of X_i and β units of Y_i. Finally, assume that the relationships between market goods and characteristics are determined ex-

[3] The implications of considering "warm-glow" motives for private provision are discussed later in the paper.

ogenously and are known by all individuals.[4] In what follows, we will consider Nash equilibria of the game in which each individual chooses how to allocate wealth among purchases of the three goods.

To maintain the most interesting case, whereby c_i, d_i, and g_i are all viable goods, further assumptions about the green technology are necessary.

Technology assumption. (i) $0 < \alpha < 1$, (ii) $0 < \beta < 1$, and (iii) $\alpha + \beta \geq 1$.

This assumption implies that c_i is the least-cost way to obtain X_i only (part i), d_i is the least-cost way to obtain Y_i only (part ii), and g_i is weakly the least-cost way to obtain both X_i and Y_i (part iii).

Each individual takes the behavior of others to be exogenous. Specifically, individual i takes the contributions of others, $Y_{-i} \equiv \sum_{j \neq i} Y_j$, as given. We can write individual i's utility maximization problem with choices over market goods:

$$\max_{c_i, d_i, g_i} U_i(X_i,\ Y_i + Y_{-i})$$

$$\text{subject to } X_i = c_i + \alpha g_i,\ Y_i = d_i + \beta g_i,\ c_i + d_i + g_i \leq w_i. \quad \text{(P1)}$$

The solution to (P1) need not be unique in the special case $\alpha + \beta = 1$. This follows because any bundle of characteristics involving strictly positive quantities of X_i and Y_i could be obtained at the same cost with an infinite number of goods bundles. The solution to (P1) will be unique, however, if $\alpha + \beta > 1$. Moreover, in this case, an individual will never choose both $c_i > 0$ and $d_i > 0$. For if this were to occur, the same bundle of X_i and Y_i could be obtained at a lower cost by increasing g_i and reducing c_i and d_i.

It is useful to write (P1) with choices over characteristics rather than goods. Figure 1 illustrates two possibilities for the budget set in terms of (X_i, Y). If $\alpha + \beta = 1$, the frontier is linear (the dashed segment *BM*); if $\alpha + \beta > 0$, the frontier is piecewise linear (segment *BEM*). The kink point E is the allocation at which all income is spent on g_i. Define $\varphi \equiv (1 - \alpha)/\beta$ and $\gamma \equiv (1 - \beta)/\alpha$. Note that $1/\varphi$ is the price ratio p_X/p_Y on segment *EM*, where the individual makes no direct donation to the public good (i.e., $d_i = 0$). Furthermore, γ is the price ratio on segment *BE*, where the individual purchases none of the pure private

[4] This setup is equivalent to a standard linear characteristics model (see Deaton and Muellbauer 1980), except that characteristic Y is a public good.

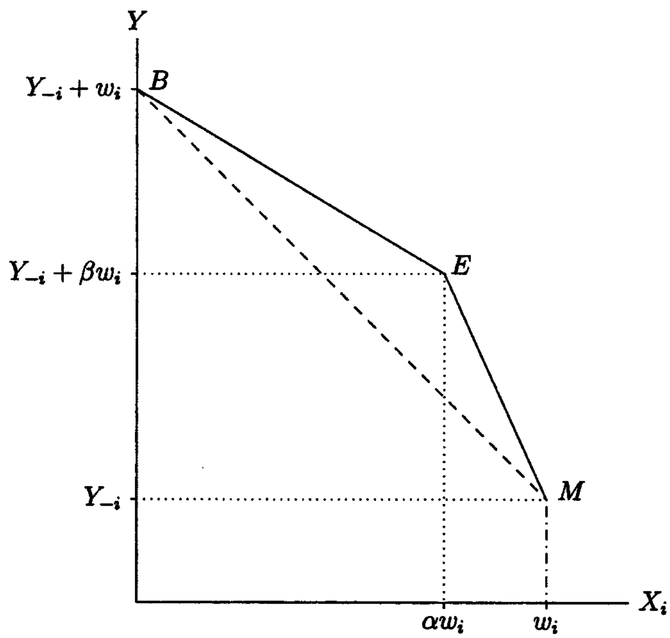

FIG. 1.—Budget frontiers in characteristics space

good (i.e., $c_i = 0$). In either case the budget set is convex, and (P1) can be rewritten as

$$\max_{X_i, Y_i} U_i(X_i,\ Y_i + Y_{-i})$$

$$\text{subject to } X_i + \varphi Y_i \leq w_i,\ \gamma X_i + Y_i \leq w_i. \tag{P2}$$

Equivalently, the individual's problem can be written with a choice over the aggregate level of Y:

$$\max_{X_i, Y} U_i(X_i,\ Y)$$

$$\text{subject to } X_i + \varphi Y \leq w_i + \varphi Y_{-i},\ \gamma X_i + Y \leq w_i + Y_{-i},\ Y \geq Y_{-i}. \tag{P3}$$

Writing the problem in this way yields the individual's "full-income" budget constraints, which account for personal income plus the value of public-good spill-ins provided by others. Full income is equivalent to Becker's (1974) notion of "social income."

The solution to (P3) will be an allocation somewhere along the frontier *BEM*, with the linear frontier *BM* (no kink) being a special case. We can express the solution in terms of the individual's demand for Y.

Suppressing notation for w_i, let the function $f_i(Y_{-i}, \alpha, \beta) = Y$ denote the unique solution to (P3).

The final assumption is standard in models of privately provided public goods. It simply assumes that both the private and public goods—or characteristics X_i and Y in this case—are normal. Formally, the assumption can be stated as follows.

Normality assumption. Consider two different allocations at which individual i's marginal rates of substitution are equal: $\text{MRS}_i(X_i', Y') = \text{MRS}_i(X_i'', Y'')$. Then if $Y'' - Y' > 0$, there exists a constant ϵ such that $X_i'' - X_i' \geq \epsilon > 0$ for all i.

A useful implication of the normality assumption is that $0 < \partial f_i(\cdot)/\partial Y_{-i} \leq 1$. We can interpret this derivative as the slope of individual i's full-income expansion path with respect to changes in spill-ins. Every individual's demand for Y is increasing in spill-ins. At interior solutions the slope is strictly less than and bounded away from one because a minimum amount of income (required by ϵ) must be spent on X_i. The slope will equal one at the kink point E, where all income is spent on the green good, or the corner M, where all income is spent on the private good. The corner solution occurs if spill-ins are sufficiently large that the individual free-rides completely and $f_i(Y_{-i}, \alpha, \beta) = Y_{-i}$. Note that a solution to this equality is guaranteed because $\partial f_i(\cdot)/\partial Y_{-i}$ is bounded away from one at interior solutions.

The first proposition establishes the existence and uniqueness of a Nash equilibrium.

Proposition 1. There exists a unique Nash equilibrium in terms of (X_i^*, Y_i^*) for all i; if $\alpha + \beta > 1$, then the equilibrium choices (c_i^*, g_i^*, d_i^*) are also unique.

The Appendix contains a proof that draws on Cornes and Hartley's (forthcoming) technique for analyzing aggregative public-good games. For future reference, let $Y^*(\alpha, \beta)$ denote the unique level of aggregate equilibrium provision. As we will see, the special case $\alpha + \beta = 1$ provides a useful benchmark for understanding the potential effects of green markets on equilibrium outcomes.

III. Green Markets and Equilibrium Provision

This section considers the different ways in which green markets can affect equilibrium provision of Y. While intuition might suggest that introducing a green market or improving its technology will increase provision of environmental quality, we will see that equilibrium provision may in fact decrease or remain unchanged.

Let us begin with the special case in which $\alpha + \beta = 1$. In this case, the utility maximization problem specified in (P2) is equivalent to the one that emerges in a market scenario in which g_i is not available, that is, when individuals can make choices only between c_i and d_i. This is equivalent to the setup in the standard model for a privately provided public good. If we let $Y^*(0, 0)$ denote the equilibrium provision that would emerge without a green market, the fact that (P2) remains unchanged proves the following result.

PROPOSITION 2. If $\alpha + \beta = 1$, then $Y^*(0, 0) = Y^*(\alpha, \beta)$.

Proposition 2 states that introducing a green market based on a technology satisfying $\alpha + \beta = 1$ will have no effect on equilibrium provision of the public good. A further implication worth highlighting is that different green technologies satisfying $\alpha + \beta = 1$ will produce the same level of provision, and the level will be no different than if there were no green market at all.

The implications of proposition 2 are important because they apply to cases in which the green good is simply a bundled commodity. Such bundling will arise, for example, if firms in a perfectly competitive market donate a proportion of their revenues (i.e., β) to a public good that is otherwise unrelated to production of the firm's good or service. In such cases, which are increasingly common, neutrality of the green market will occur regardless of the level of β. This follows because, rather than purchase g_i, individuals are equally content to purchase α units of c_i and to make a direct donation d_i of β units. Note that an implicit assumption in this argument is that the normalized prices account for potential differences in the transactions costs associated with the two different alternatives.

Let us now turn to the more general case in which $\alpha + \beta > 1$. Two simple examples are useful to demonstrate different possibilities. Assume that the economy consists of two individuals with identical endowments $w_i = w$ and preferences according to the constant elasticity of substitution utility function $U_i = (X_i^\rho + Y^\rho)^{1/\rho}$. Let $w = 100$ and $\rho = .3$. Without a green market, the Nash equilibrium will be symmetric, and it is straightforward to solve for each individual's level of provision $Y_i^* = w/3$ and the aggregate level of provision $Y^* = 2w/3 \cong 67$. This level of Y serves as the reference point for the following examples that include a green market.

EXAMPLE A. The green technology is characterized by $\alpha = \beta = .6$. Solving for each individual's equilibrium level of provision yields $Y_i^A = \varphi^{\pi-1}w/(2 + \varphi^\pi)$, where $\pi = \rho/(\rho - 1)$. Substituting in the numerical values, we have $Y^A = 2Y_i^A \cong 112$. Figure 2*a* illustrates this example's *increase* in environmental quality from Y^* to Y^A.

EXAMPLE B. This example differs only with respect to the green technology, which now favors production of the private characteristic with $\alpha = .9$ and $\beta = .3$. Solving for each individual's level of provision yields $Y_i^B = w/(1 + 2\gamma^\pi)$. Substituting in the numerical values, we have

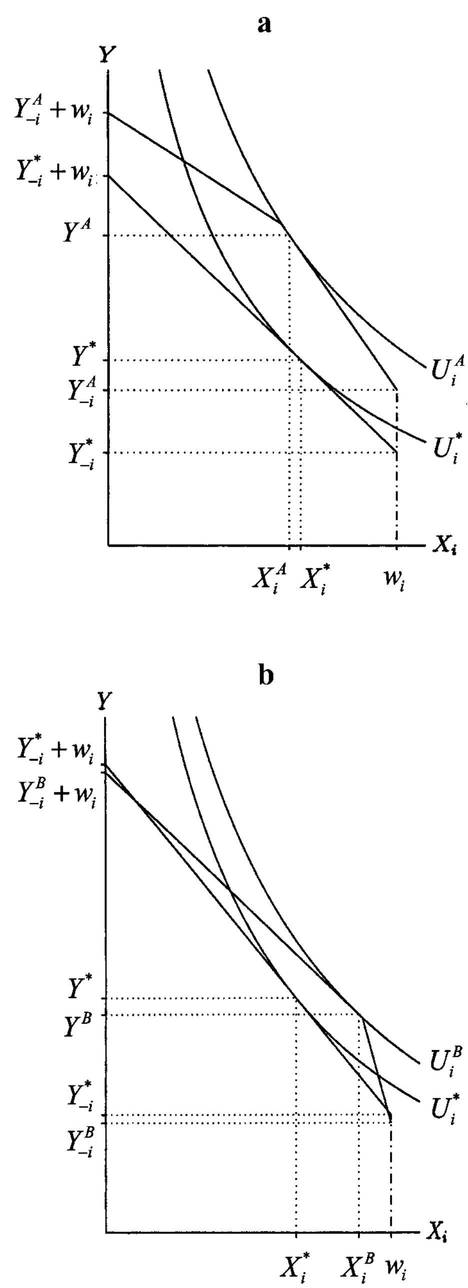

FIG. 2.—*a*, example A; *b*, example B

$Y^B = 2Y_i^B \cong 62$. Figure 2*b* illustrates this example's *decrease* in environmental quality from Y^* to Y^B.

To gain intuition for these examples, it is useful to think of introducing a green market as having two effects on each individual. The first is a *price effect* from a change in the implicit prices of X_i and Y, with Y_{-i} held constant. The second is a *spill-in effect* from a change in the level of Y_{-i}. Both effects contribute to changes in each individual's demand for Y, which in turn influence changes in the equilibrium level of Y.

In example A, both the price effect and the spill-in effect stimulate demand for Y. Both individuals move to the lower portion of the new budget frontier and therefore face a lower relative price for Y (from unity to $\varphi < 1$). This price effect stimulates demand for Y, which encourages provision. Then, increased provision by one individual increases Y_{-i} for the other. This generates a positive spill-in effect because an increase in Y_{-i} increases full income and Y is normal. The overall result is an *increase* in the equilibrium level of Y.

Example B, in contrast, illustrates a case in which both the price effect and the spill-in effect decrease demand for Y. Individuals move to the upper portion of the new budget frontier and therefore face a lower relative price for X_i (from unity to $\gamma < 1$). This price effect decreases demand for Y because in this example, with $\rho \in (0, 1)$, it is the case that Y is a gross substitute for X. Then, as individuals begin to reduce their provision, spill-ins Y_{-i} are reduced as well. This generates negative spill-in effects, and the overall result is a *decrease* in the equilibrium level of Y.

The price effect is particularly important for explaining the qualitative difference between the two examples. The price effect stimulated demand for Y in example A, whereas it diminished demand for Y in example B. It is worth emphasizing, therefore, that the negative price effect in example B is the result of Y being a gross substitute for X. If Y is a gross complement for X—meaning that demand for Y is decreasing in p_X—the price effect of introducing a green good or improving its technology will always be (weakly) positive, as the following lemma states formally.

Lemma 1. If Y is a gross complement for X_i for individual i, then $f_i(Y_{-i}, \alpha, \beta) \leq f_i(Y_{-i}, \alpha', \beta')$ for all $\alpha \leq \alpha'$ and $\beta \leq \beta'$.

We can prove lemma 1 with a graphical argument. When Y_{-i} is held constant, a change in technology from (α, β) to (α', β') can only expand the individual's budget set. The new frontier resembles segment *BEM* in figure 1, and the initial frontier lies entirely inside it. A new consumption bundle on segment *BE* would imply a decrease in the price of X, whereas a new bundle on segment *EM* would imply a decrease in the price of Y. If Y is a gross complement for X, either case implies an increase in demand for Y. At the kink point E, demand for Y can only increase or remain the same. The final possibility is that demand for Y remains at the corner M, which is unchanged since Y_{-i} is held constant. We have thus shown that demand for Y cannot decrease and that lemma 1 must hold.

The argument for lemma 1 carries over to an equilibrium result as well. If every individual is willing to contribute a greater amount of Y when a green good becomes available, then introducing a green market will increase equilibrium provision of Y. Similarly, if improving the technology of a green good stimulates each individual's demand for Y, it will increase equilibrium provision. Both of these results are captured in the next proposition, which is proved in the Appendix.

PROPOSITION 3. If Y is a gross complement for X_i, for all i, then $Y^*(\alpha, \beta) \leq Y^*(\alpha', \beta')$ for all $\alpha \leq \alpha'$ and $\beta \leq \beta'$.

Without the gross complement condition, green markets may cause some individuals to experience a positive price effect and others to experience a negative price effect. As a result, changes in equilibrium provision will depend on preferences, the distribution of income, and the green technology. Examples A and B showed that if Y is a gross substitute for X_i, then introducing a green market or improving its technology can either increase or decrease provision of the public good. The possibility of diminished environmental quality arises because a change in the green-market technology affects the implicit prices of private consumption and environmental quality.

IV. Green Markets in a Large Economy

A well-known result is that group size influences equilibrium outcomes for private provision of public goods. This section addresses the question of how group size influences green-market effects on equilibrium provision of Y. There are two main results. First, if the economy is sufficiently large—in terms of the number of individuals—then green markets based on technologies satisfying $\alpha + \beta > 1$ will crowd out all direct donations. Second, green-market effects on equilibrium provision are unambiguous in a sufficiently large economy: introducing a green market or improving its technology will never decrease aggregate provision of Y.

To begin, we need to establish the mechanism for changing the size of the economy. A useful approach is to add individuals through a sequence of replica economies.[5] Start with an economy composed of n individuals, where each individual represents a unique type. Then, to

[5] The approach described here is commonly used to prove core equivalence theorems (e.g., Debreu and Scarf 1963), but it has also been used to study the effects of group size in models of privately provided public goods (e.g., Roberts 1976).

increase the size of the economy, replicate the initial economy q times. Thus economy q consists of $N = nq$ individuals and has q individuals of each type in the initial economy (i.e., with the same preferences and endowment).[6]

The next proposition, which is proved in the Appendix, applies to all green technologies for which joint production is more efficient than separate production (i.e., $\alpha + \beta > 1$). It states that if the economy is sufficiently large and the green good is available, then no individual will make a direct donation.

PROPOSITION 4. If $\alpha + \beta > 1$, then there exists a number of replicas $\hat{q} \geq 1$ such that, for all $q \geq \hat{q}$, equilibrium direct donations in economy q are zero for all types; that is, $d_i^*(q) = 0$ for all i.

The intuition for proposition 4 follows directly from the pure public good model. We know from the standard model that an increase in the number of individuals who are willing to contribute will increase aggregate provision. But, at the same time, each contributor has a greater incentive to free-ride and therefore contributes less. The same reasoning applies with a green market, but with the added result that each individual's lower provision will eventually imply no direct donations, as each individual chooses to provide a lower quantity of Y through the green good only, if at all.

What are the green-market effects on equilibrium provision of Y in a large economy? We have already shown that green markets will have no effect on equilibrium provision if $\alpha + \beta = 1$. More generally, if $\alpha + \beta > 1$, we have also shown that green markets can either increase or decrease aggregate provision of Y. We now have a further result, which implies that green-market effects in a sufficiently large economy are unambiguous.

PROPOSITION 5. There exists a number of replicas $\hat{q} \geq 1$ such that, for all $q \geq \hat{q}$, equilibrium provision will satisfy $Y^*(\alpha, \beta, q) \leq Y^*(\alpha', \beta', q)$ for all $\alpha \leq \alpha'$ and $\beta \leq \beta'$.

Proposition 5 states that in any sufficiently large economy, introducing a green market or improving its technology will not decrease aggregate provision and will generally increase it. The proof is a special case of the results proved already. An actual improvement in the technology implies $\alpha' + \beta' > 1$. We know from proposition 4 that with technology (α', β') there exists a $\hat{q}$ such that, for all $q \geq \hat{q}$, no individual in the economy will make a direct donation. Thus it is sufficient to show that if no individual makes a direct donation with the new technology, then equilibrium provision must (weakly) increase. But this result follows

[6] An alternative approach for increasing the size of the economy is to specify a probability density function from which individuals are drawn (e.g., Andreoni 1988). While this approach is useful for establishing asymptotic results as $n \to \infty$, adding a sequence of replica economies is useful for establishing finite results as well.

directly from lemma 1 and proposition 3. It is straightforward to verify that lemma 1 holds for every individual without the gross complement condition if there are no direct donations with the new technology. Consequently, the proof of proposition 3 goes through without the gross complement condition as well, and this proves proposition 5.

Proposition 5 shows the interaction between the number of individuals, direct donations, and aggregate provision of the public good. Individuals in larger economies are less likely to make direct donations, choosing instead to provide environmental quality through consumption of the green good. Yet, in larger economies, an improvement in the green technology is more likely to increase total provision of environmental quality.

V. Social Welfare

Introducing or improving a green technology expands each individual's choice set and the production possibilities, with fixed provision of the public good by others held fixed. Nevertheless, green markets need not result in a Pareto improvement. This section uses a series of examples to show the possible consequences on social welfare of introducing or improving a green technology. It is shown that even if total provision of the public good increases, utility for some individuals may fall. Moreover, if total provision of the public good decreases, utility for all individuals may fall.

Let us begin with cases in which the green market increases aggregate provision of Y. Figure 2*a* illustrates a case with a Pareto improvement. If we again consider the notions of a price effect and a spill-in effect, it is clear that both the lower price of Y and the increased spill-ins result in positive income effects, which cause the increase in utility for both individuals.

More generally, heterogeneity in individual preferences and endowments can generate differences in the sign and magnitude of the green market's price effect, and this, in turn, can induce some individuals to provide more Y and others to provide less. Consequently, despite a net increase in Y and lower implicit prices, some individuals can be made worse off because of a negative income effect from a sufficiently large reduction in their spill-ins. In other words, the green market can shift the burden of provision such that someone is worse off. It can be shown that such a scenario is possible regardless of whether Y is a gross substitute for or complement to X_i. An example of each is provided in the Appendix.[7]

[7] In a related model, Cornes and Sandler (1989) find that a technological improvement in production of the public good will always increase provision and result in a Pareto

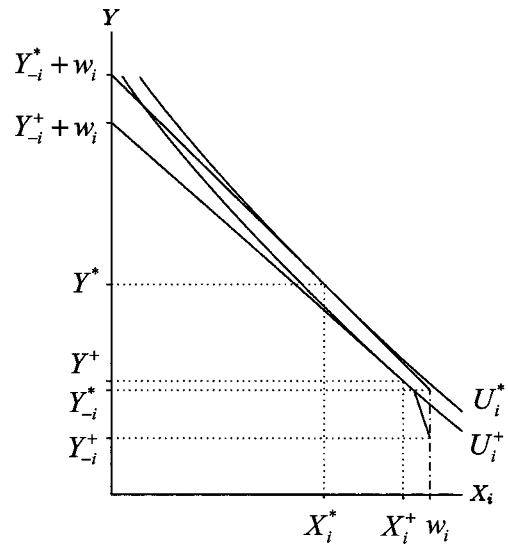

FIG. 3.—Green market decreases both provision and utility

Now consider cases in which the green market causes a decrease in aggregate provision of Y. Figure 2b illustrates another case with a Pareto improvement. Because both individuals provide less, the decreased spill-ins generate negative income effects, but these are more than offset by the positive income effects from the lower price of X_i. Thus utility increases for both individuals, despite the decrease in Y.

Perhaps the most counterintuitive possibility is one in which the green market makes every individual worse off. Figure 3 illustrates an example ($w = 100, \alpha = .95, \beta = .15$, and $\rho = .88$). Here again, equilibrium provision decreases (from Y^* to Y^+), and the reduced spill-ins generate negative income effects for both individuals. But in this case the positive income effects from the lower price of X_i are not large enough to be offsetting. As a result, utility declines for both individuals (from U_i^* to U_i^+).

When is a green market likely to reduce welfare? Assume that there are n individuals with identical preferences and endowments, and that they make direct donations even when the green good is available. In this case, the analysis has a close parallel with Cornes and Sandler's (1989) study of the possibility for immiserizing growth in a public-goods

improvement. This scenario is closely related to the case here in which no individual makes a direct donation. While proposition 5 shows that the level of provision will increase, the first example in the Appendix shows that a Pareto improvement need not follow. The difference arises because Cornes and Sandler's result applies only to an economy of n identical individuals.

economy. Their approach can be adapted to derive equilibrium conditions under which $dU/d\alpha < 0$ and $dU/d\beta < 0$. These conditions, which are derived in the Appendix, are as follows:

$$\frac{dU}{d\alpha} < 0 \Leftrightarrow \gamma X_i < \frac{\alpha f_{\alpha}(Y_{-i}, \alpha, \beta)}{f_{Y_{-i}}(Y_{-i}, \alpha, \beta) - [n/(n-1)]} \quad (1)$$

and

$$\frac{dU}{d\beta} < 0 \Leftrightarrow X_i < \frac{\alpha f_{\beta}(Y_{-i}, \alpha, \beta)}{f_{Y_{-i}}(Y_{-i}, \alpha, \beta) - [n/(n-1)]}. \quad (2)$$

The normality assumption implies that the denominator on the right-hand side of the inequalities is negative; therefore, the numerator must be negative in order to satisfy the condition. Thus Y being a gross substitute for X_i is necessary although not sufficient. The larger the magnitude of the price effect (f_{α} or f_{β}), the more likely it is that the conditions will be satisfied. This is consistent with a greater elasticity of substitution between X_i and Y, as the contrast between the indifference curves in figure 2*b* and figure 3 suggests. Other important features that make immiserization more likely are a larger spill-in effect, captured by $f_{Y_{-i}}$; green technologies that are more favorable to production of X_i, as captured by a larger α; lower initial consumption of X_i, which implies that the income effects of the green market's decrease in the price of X_i will be less significant; and a larger n, which implies more individuals who reduce their provision and decrease each individual's spill-ins.

More generally, with heterogeneous preferences and endowments, green markets will have a greater impact when individuals are more responsive to changes in the price of characteristics. Green markets are more likely to be welfare-improving if they encourage provision of the public good. In contrast, the welfare implications are more likely to be negative if green markets promote substitution away from the public good. The possibility for such immiserizing effects is more likely when the green technology favors production of the private characteristic and when consumption of the private characteristic starts at relatively low levels.

VI. Conclusion

This paper analyzes a new choice setting for private provision of a public good. The model applies when there is an impure public good whose characteristics are also available separately, through a private good and a pure public good. The model is applied in particular to green markets, which offer impure public goods through joint production of a private good and an environmental public good. Green markets fit the model because in addition to the green good, consumers typically have opportunities to consume a conventional version of the good and to make a direct donation to the associated environmental public good.

The paper has four main results. First, introducing or improving a green technology can either increase or decrease private provision of the associated environmental public good. Second, the effect that a green technology has on the level of private provision depends heavily on whether the public good is a complement to or substitute for private consumption. Third, in a sufficiently large economy, introducing or improving a green technology will crowd out all direct donations, yet always increase the level of provision. Fourth, green technologies can be either welfare-improving or immiserizing, despite the fact that they expand the consumer choice set and increase the production possibilities.

How might the results change if private provision were motivated in part by warm glow? With warm-glow preferences, individual utility functions are specified as $U_i = U_i(X_i, Y, Y_i)$, implying a distinct private benefit from one's own level of provision (Andreoni 1989, 1990). The only results that may differ are those for a sufficiently large economy, which may no longer hold. If the warm-glow motive is strong enough, individuals may continue to make direct donations, regardless of how much Y is provided by others. Accordingly, it is possible for the green market's price effect to discourage provision of Y even in a very large economy.

The increased availability of impure public goods in the economy has both positive and normative consequences. In the context of green markets, this paper demonstrates how these consequences can be counterintuitive. Although green markets are promoted to improve environmental quality and promote social welfare, their actual effects may be detrimental to both. These results, along with the conditions sufficient to rule then out, provide new insight into the potential advantages and disadvantages of green markets as a decentralized mechanism of environmental policy. The results also apply more generally to other situations—such as socially responsible investing and charitable fund-raising through commercial activities—in which the joint products of an impure public good are also available separately.

Appendix

Proof of Proposition 1

Fix α and β and suppress them. For each i, define $a_i = f_i(0)$ and b_i implicitly with $\text{MRS}_i(w_i, b_i) = 1/\varphi$. Thus a_i is demand for the public good by individual i if provision by others is zero; if provision by others is at least b_i, then individual i spends w_i entirely on the private good. For $Y \geq a_i$, define $r_i(Y)$ implicitly with $f_i(Y - r_i(Y)) = Y$. Thus $Y_{-i} = Y - r_i(Y)$ is the level of provision by others that leads individual i to consume Y and provide $Y_i = r_i(Y)$. By the implicit function theorem, r_i is well defined and continuous, with $r_i(a_i) = a_i$ and $r_i(b_i) = 0$. The normality assumption implies that $0 < f_i'(Y_{-i}) \leq 1$, so

$$r_i'(Y) = 1 - \frac{1}{f_i'(Y - r_i(Y))} \leq 0,$$

which holds with equality if $f_i'(Y_{-i}) = 1$. Thus $r_i(Y)$ is nonincreasing on (a_i, b_i), and $r_i(Y) = 0$ for $Y \geq b_i$.

Define $a \equiv \max\{a_i\}_{i=1}^n$ and $b \equiv \max\{b_i\}_{i=1}^n$. For $Y \geq a$, define $R(Y) \equiv \sum_{i=1}^n r_i(Y)$. Clearly, $R(Y)$ is continuous and nonincreasing, with $R(a) \geq a$ and $R(Y) = 0$ for all $Y \geq b$. Hence there is a unique Y^* that satisfies $R(Y^*) = Y^*$, and $Y_i^* = r_i(Y^*)$ specifies a unique equilibrium strategy (Y_i^*, X_i^*) for all i. Finally, if $\alpha + \beta > 1$, the mapping between (Y_i^*, X_i^*) and (c_i^*, g_i^*, d_i^*) is also unique. QED

Proof of Proposition 3

Define $r_i(Y; \alpha, \beta)$ and $R(Y; \alpha, \beta)$ as in the proof of proposition 1. The normality assumption combined with lemma 1 implies that $r_i(Y; \alpha, \beta) \leq r_i(Y; \alpha', \beta')$ for all i and $\alpha \leq \alpha'$, $\beta \leq \beta'$. Hence $R(Y; \alpha, \beta) \leq R(Y; \alpha', \beta')$, and it follows immediately that $Y^*(\alpha, \beta) \leq Y^*(\alpha', \beta')$. QED

Proof of Proposition 4

Fix α and β and suppress them. Define $r_i(Y)$ as in the proof of proposition 1, and define $R(Y) = \sum_{i=1}^n r_i(Y)$ for only the initial economy of n individuals. The Nash equilibrium $Y^*(q)$ with q replicas must satisfy $qR(Y^*(q)) = Y^*(q)$. It follows, because $R(Y)$ is nonincreasing, that $Y^*(q)$ is strictly increasing in q. If type i individuals make a direct donation in equilibrium, they must be at an interior solution with $r_i(Y^*(q)) > \beta w_i$. The normality assumption implies that $r_i(Y)$ is strictly decreasing at interior solutions and that it will eventually hit βw_i. Hence there exists a $\hat{q}$ such that $r_i(Y^*(q)) \leq \beta w_i$ for all i types and $q \geq \hat{q}$, and this implies no direct donations. QED

Examples in Which $Y^* < Y^+$ and $U_i^* > U_i^+$ for Some i

Consider an example in which Y is a gross substitute for X_i. There are two individuals with endowments $w_1 = 160$ and $w_2 = 100$. Preferences are given by $U_i = (X_i^{\rho_i} + Y^{\rho_i})^{1/\rho_i}$, where $\rho_1 = .8$ and $\rho_2 = .1$. Without a green market, each individual's private provision is $Y_i^* = (2w_i - w_j)/3$, and private consumption is $X_i^* = w_i - Y_i^*$. Aggregate provision is $Y^* = (w_1 + w_2)/3 \cong 87$. With a green market characterized by $\alpha = .2$ and $\beta = .9$, neither individual makes a direct donation, and private provision is $Y_i^+ = (\varphi^{\pi_i - 1} w_i A_j - \varphi^{\pi_j - 1} w_j)/(A_j A_i - 1)$, where $\pi_i = \rho_i/(\rho_i - 1)$ and $A_i = 1 + \varphi^{\pi_i}$. Private consumption is $X_i^+ = w_i - \varphi Y_i^+$, and aggregate provision is $Y^+ = [\varphi^{\pi_i + \pi_j - 1}(w_i + w_j)]/(A_j A_i - 1) \cong 112$. With this example, it is straightforward to verify numerically that $Y_1^* < Y_1^+$ and $Y_2^* > Y_2^+$, and that $U_1^* > U_1^+$ and $U_2^* < U_2^+$.

Now consider an example in which Y is a gross complement to X_i. Both individuals have identical preferences with $\rho = -1$. Endowments differ such that $w_1 = 100$ and $w_2 = 200$. Without a green market it remains that $Y_i^* = (2w_i - w_j)/3$ and $X_i^* = w_i - Y_i^*$, but now $Y^* = (w_1 + w_2)/3 = 100$. Assume that the green market is characterized by $\alpha = .8$ and $\beta = .3$. When the green good is available, it can be shown that $d_1^+ = 0$ but $d_2^+ > 0$. Accordingly, $Y_1^+ = (\varphi^{\pi-1}w_1B_2 - \gamma^{-\pi}w_2)/(B_2A_1 - 1)$ and $Y_2^+ = (\gamma^{-\pi}w_2A_1 - \varphi^{\pi-1}w_1)/(B_2A_1 - 1)$, where $B_2 = 1 + \gamma^{-\pi}$. Private consumption is $X_1^+ = w_1 - \varphi Y_1^+$ and $X_2^+ = (w_2 - Y_2^+)/\gamma$. Moreover, $Y^+ = [\varphi^{\pi}\gamma^{-\pi}(\varphi^{-1}w_1 + w_2)]/(B_2A_1 - 1) \cong 111$. In this case, it is straightforward to verify numerically that $Y_1^* < Y_1^+$ and $Y_2^* > Y_2^+$, and $U_1^* > U_1^+$ and $U_2^* < U_2^+$.

Derivations of Expressions (1) and (2)

We have assumed that all individuals have identical preferences and endowments, and that they make direct donations when the green good is available. Starting at an initial equilibrium, which will be symmetric, we can solve for $dU/d\alpha$. The initial equilibrium must satisfy $Y = f([(n-1)/n]Y, \alpha, \beta)$. Taking the differential of this function, holding β constant, and rearranging yields

$$\frac{dY}{d\alpha} = \frac{nf_{\alpha}(\cdot)}{n - (n-1)f_{Y_{-i}}(\cdot)}.$$

Taking the differential of the utility function and using the fact that $\mathrm{MRS}(X_i, Y) = \gamma$ yields $dU = U_{X_i}[dX_i + (1/\gamma)dY]$. Taking the differential of the binding budget constraint, holding β and w constant, and rearranging yields

$$dX_i = \frac{X_i d\alpha}{\alpha} - \frac{dY_i}{\gamma}.$$

Substituting this expression into the expression for dU, dividing by $d\alpha$, and substituting in the expression for $dY/d\alpha$ yields

$$\frac{dU}{d\alpha} = U_{X_i}\left\{\frac{X_i}{\alpha} + \frac{f_{\alpha}(\cdot)}{\gamma([n/(n-1)] - f_{Y_{-i}}(\cdot))}\right\}.$$

The inequality in (1) follows from noting that $dU/d\alpha < 0$ if and only if the term in braces is less than zero.

We can solve for $dU/d\beta$ following the same steps, but letting β change and holding α constant. This yields

$$\frac{dU}{d\beta} = \frac{U_{X_i}}{\gamma}\left\{\frac{X_i}{\alpha} + \frac{f_{\beta}(\cdot)}{[n/(n-1)] - f_{Y_{-i}}(\cdot)}\right\}.$$

Here we can see, as stated in expression (2), that $dU/d\beta < 0$ if and only if the term in braces is less than zero.

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