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A New Stocking Guide Formulation Applied to Eastern White Pine

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ABSTRACT. A procedure is described for incorporating stand age, or stage in development, into the widely used stocking guides based on the crown competition factor (CCF). A simple model that predicts dbh of individual trees from crown projection area and total height is derived from a three-dimensional relationship between cumulative foliage production (crown volume) and total volume of the main stem. Procedures employed by previous researchers to prepare CCF-based stocking guides are then used to convert this individual-tree model to stand basal areas and numbers of trees per unit area at successive stages in development denoted by stand heights. An equation relating crown width to crown length and height is derived and used to predict growth under conventional and low-density thinning schedules for eastern white pine. An example shows that complete crown closure can occur well below the CCF-based B level if early thinnings are heavy enough to permit development of large crowns. Because CCF-based stocking guides assume that the crown width-dbh relationship is independent of height, they may, in some cases, overestimate the number of trees required for full crown closure. FOR. SCI. 33(2):469–484.

ADDITIONAL KEY WORDS. *Pinus strobus* L., stand density, crown competition factor, crown shape, crown-stem relationships, thinning schedules.

WHEN PLANNING STAND DENSITY management regimes, foresters need an objective means of determining how much growing space must be allocated to individual trees to ensure that they reach the desired size at a specific time in stand development. Because stem wood is formed from sugars produced in the leaves, the most logical basis for predicting tree growth, short of counting the leaves themselves, is some parameter of crown size that serves as a surrogate for foliar biomass. If the relationship between crown size and wood production of individual trees can be quantified, it provides a biologically sound method of estimating the equivalent stand-level parameters (basal area and number of trees at a given age or height) that comprise the essential elements of stocking.

This paper presents a new approach to this important silvicultural problem of density management. A simple three-dimensional relationship between crown size and bole volume is derived and used to modify the widely used stocking guides (e.g., Gingrich 1967, Leak et al. 1969, Frank and Bjorkbom 1973, Philbrook et al. 1973, Roach 1977, Safford 1983) based on the crown competition factor (Krajicek et al. 1961) which are independent of site and age. The resulting stocking equation includes stand height, which can readily be translated into age or passage of time using site index

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curves. Incorporating height into the stocking equation provides a simple means of predicting how many trees can be grown to a certain dbh at various points in the rotation (defined by different tree heights) or conversely, to determine how long it will take a fixed number of crop trees to reach the desired size. This obvious advantage of including height is not new. It was recognized by Wilson (1946) and Briegleb (1952) and has subsequently been included into density management diagrams for western species (e.g., Drew and Flewelling 1979, McCarter and Long 1986). However, this has remained a weakness of the guides based on the crown competition factor that do not include height and cannot predict changes over time.

Methods

Model Derivation

First, we assume that total bole volume is proportional to the total amount of foliage produced by a tree during its entire life:

$$BV = k_0 TCV^{k_1} \tag{1}$$

where BV = total bole volume, TCV = total crown volume, k_0 = a constant related to foliar efficiency, and k_1 allows efficiency to vary with crown size. The expression TCV is an integrated measure of all space that is now, or was formerly, occupied by leaves. It is similar to the expression used by Berlyn (1962) that models current crown volume, but also includes all space below the current live-crown base that was once occupied by foliage and living branches, which can no longer be measured. Its dimensions are established by two variables: tree height, which depends mainly on site and age; and branch length, which is determined primarily by competition from surrounding trees after crown closure.

For many conifers, growth of a branch is a function of its distance from the leader (Mitchell 1975). As the crown develops, it assumes the shape of a geometric solid with a volume given by the formula:

$$V = k \pi R^2 H$$

where R = radius of the base (assumed to be circular), H = height, and k is a parameter that defines the specific solid. For example, if $k = 1/3$, the solid is a cone; if $k = 0.5$, it is a paraboloid. Paraboloids have been used to model the crowns of *Quercus rubra* L. and *Fraxinus americana* L. (Holsoe 1948), *Liriodendron tulipifera* L. (Holsoe 1951), *Populus deltoides* Bartr. (Berlyn 1962), and *Acer saccharum* Marsh. (Allen 1976). In plantation-grown *Pinus resinosa* Ait., Stiell (1962) found a nearly perfect correlation between foliage weight and crown volume computed from the paraboloid formula. Crowns of other conifers such as *Picea glauca* (Mitchell 1969) and *Abies balsamea* (Honer 1971) have been successfully modeled by shapes more closely resembling a cone. In general, the live crown volume (above the widest point) of many species can be estimated from:

$$UCV = k_2 \pi CR^2 CL \tag{2}$$

where UCV = upper crown volume, k_2 = a shape parameter unique to the species, CR = average crown radius, and CL = crown length, measured from the widest point of the crown to the top of the tree (Figure 1).

We further assume that if the crown radius has not begun to recede from competition, the space formerly occupied by now-dead branches and the associated leaves (plus any living branches below the widest point of the crown) can be approximated by a similar solid inverted with its base at the widest point of the crown and its apex at the ground:

$$LCV = k_2 \pi CR^2 (H - CL) \tag{3}$$

where LCV = lower crown volume (below the widest point), H = total tree height, and other variables are as above.

Total crown volume (TCV) in (1) can now be estimated simply by adding (2) to (3) and simplifying:

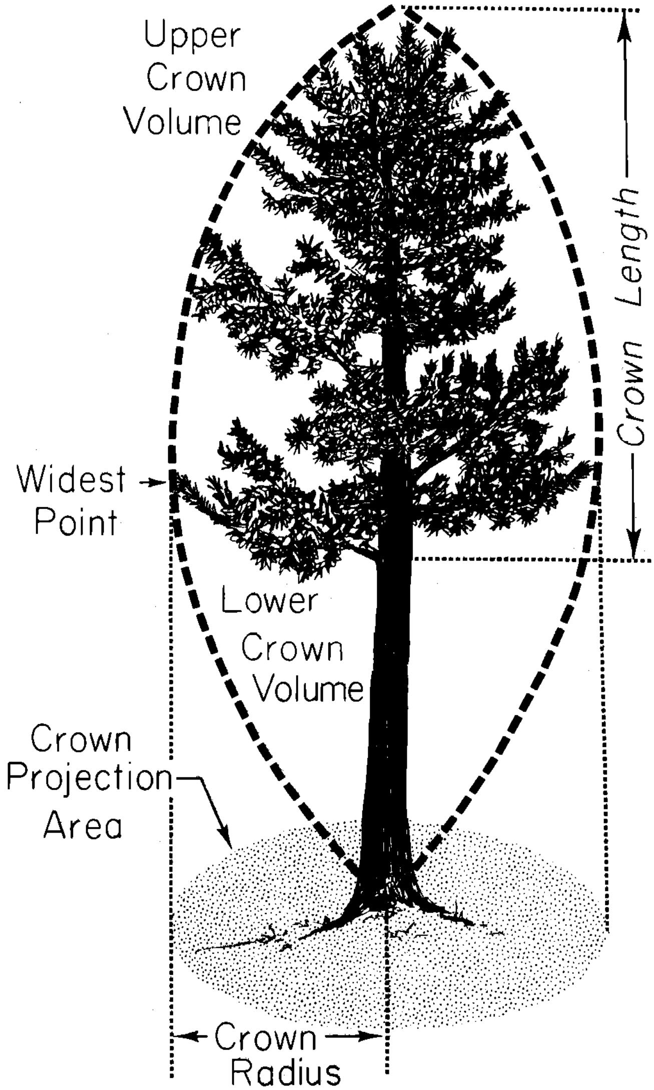

FIGURE 1. Schematic of white pine crown showing variables used to derive stocking equation.

$$TCV = k_2 \pi CR^2 [CL + (H - CL)],$$

or simply

$$TCV = k_2 \pi CR^2 H \quad (4)$$

Since πCR^2 = vertical crown projection area (CPA), (4) can be rewritten as:

$$TCV = k_2 \, CPA \, H \quad (5)$$

If UCV and LCV have the same geometric shape [i.e., k_2 is the same in equations (2) and (3)], neither CL nor height to the widest point of the crown appear in the final model, even though they enter logically into its derivation.

Mensurationists have used many different equations to model total bole volume. One of the most widely applied and flexible forms was originally proposed by Schumacher and Hall (1933):

$$BV = a \, D^b \, H^c \quad (6)$$

where BV = total volume of the bole, D = diameter at breast height (dbh), and a, b and c are regression parameters.

Equations (5) and (6) provide expressions for both TCV and BV in terms of conventional tree dimensions. These variables can be calculated for individual trees and used to fit equation (1) directly, but it is subject to criticism because height appears on both sides of the equation. A better approach is to substitute the right-hand sides of (5) and (6) into (1), and solve for D. It can be shown through algebraic manipulation (Appendix A) that (1) reduces to a simple model that predicts the dbh a tree will reach if its crown grows to cover a certain area at a given height:

$$D = b_0 \, CPA^{b_1} \, H^{b_2} \quad (7)$$

where b_i are parameters estimated by regression analysis from data on forest-grown trees that have developed over a range in stand densities.

If the geometric arrangement of crowns in a stand with full canopy cover is specified, CPAs can be converted to stand densities (trees per unit area) or equivalent between-tree spacings that meet certain dbh goals. Including height permits data from a range of site and age classes to be combined, and more importantly, allows the relationship between growing space and tree size to be evaluated at any point during the rotation. This feature overcomes a major drawback of the widely used stocking guides based on the crown competition factor (CCF) that are independent of age and thus do not allow stocking to be evaluated relative to this important variable.

Data Collection and Analysis

To test this model and illustrate its application, Equation (7) was fitted to data on 122 eastern white pines from five sites in southern New England. Dominant and codominant pines with symmetrical crowns were selected over the range of diameters in each stand. The smallest trees were about 4 in. dbh, 40 ft tall in a densely stocked stand; the largest were over 22 in. dbh, 80 ft tall, which had grown in isolation since a severe hurricane blew down their competitors when they were 4–6 in. dbh. On two occasions about 10 years apart, the following dimensions were measured: total height, dbh, crown radii (four perpendicular measurements, beginning with the longest radius), and heights to the widest point of the crown and to the lowest living whorl. Age ranged from 29 to 85 years; site index varied from about 55 to nearly 90 ft (50-yr base).

To stabilize the variance, data were transformed to logarithms and fitted to the linear form of Equation (7) by multiple regression. Repeat measurements were treated as separate observations. Crown projection areas were assumed to be circular, with a radius equal to the arithmetic mean of the four measurements. If the average radius had receded between measurements, the first-measurement CPA was paired with the second-measurement height and dbh when fitting the regression. This was done to prevent underestimating the total crown space formerly occupied by a few small codominants that had begun to lapse into the intermediate crown class.

Results

Regression Model

Fitting Equation (7) gave:

$$\ln D = -1.9899 + 0.2787 \ln CPA + 0.7003 \ln H \; (R^2 = 0.92)$$

or in nonlinear form:

$$D = 0.1367 \, CPA^{0.2786} \, H^{0.7003} \tag{8}$$

where D is in inches, CPA is in square feet, and H is in feet. As expected, CPA was more highly correlated with D, but H was also highly significant ($P = 0.01$) and its addition to the regression increased R^2 by over 9%. This influence of height on the D-CPA relationship is clearly evident in Figure 2, which shows that at a given CPA, taller trees have larger dbhs.

Equation (8) provides a simple, objective method for determining how much three-dimensional growing space must be allocated to individual pines to ensure that they reach a certain dbh at a specific point during the rotation. Height at a particular age is largely determined by site quality; however, CPA can take on a wide range of values depending on stand density. Thus, the challenge is to determine how wide crowns must be to achieve the desired dbh at a given height. For this purpose, (8) can be solved for CPA:

$$CPA = 1265 \, D^{3.589} \, H^{-2.514} \tag{9}$$

For example, if the goal is to grow 16-in. dbh trees at a height of 80 ft, the required $CPA = 1265 \; 16^{3.589} \; 80^{-2.514} = 436$ ft^2 per tree, which indicates that 100 trees per acre can reach this size if the crowns fully cover the area. Table 1, which shows the solution of (8) for heights of 40 through 80 ft and selected values of CPA over its observed range for each height class, illustrates the wide spectrum of dbhs that can be grown by controlling crown size in early thinnings. A similar version of this table, in which CPAs are converted to equivalent trees per acre or square spacing, is presented by Smith and Seymour (1986) in conjunction with other important considerations in deriving white pine thinning schedules.

Crown Size Limits

Maximum dbh is limited by the largest crown width that can develop at a given height. In addition to the obvious constraints imposed by competition from surrounding trees, the fundamental crown geometry of conifers is highly predictable and can be used to establish upper bounds on CPA in the absence of such competition. If appropriate data are available, crown width can be modeled as a function of crown length and total tree height. After

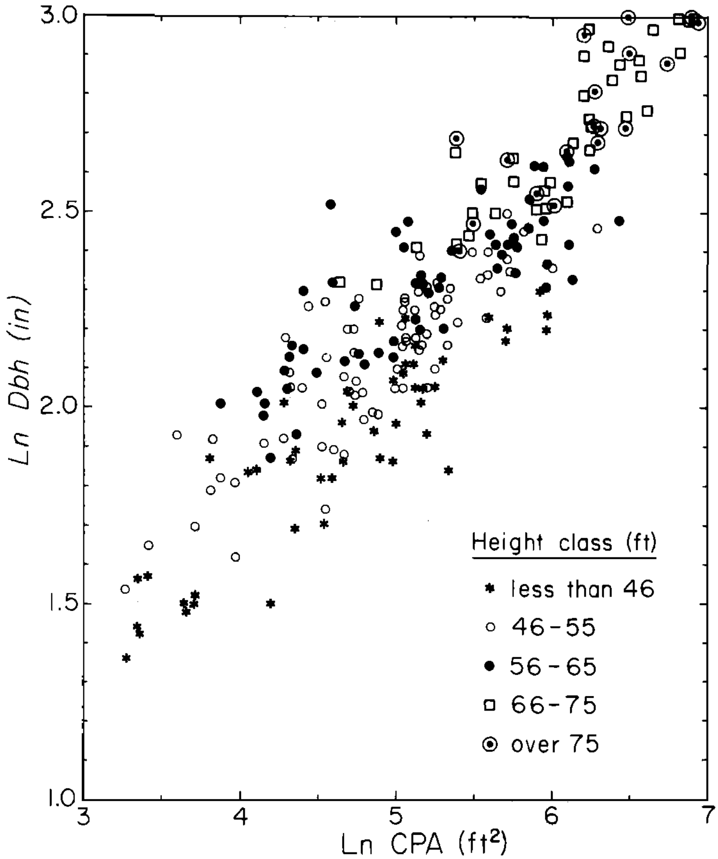

FIGURE 2. Influence of stand height on the relationship between crown projection area (CPA) and dbh showing data used to fit regression Equation (8).

preliminary screening of many combinations of CL and H, the following no-intercept model was derived:

$$CR_{max} = 0.6027\ CL - 0.009988\ CL^2 + 0.00006024\ HCL^2 \quad (10)$$
$$[R^2 = 0.63],$$

TABLE 1. Dbhs resulting from different crown sizes for eastern white pine, at heights of 40 through 80 ft. [Calculated from Equation (8) over observed range in CPA for each height class.)

Height (ft)	Crown projection area (ft^2) 25	50	100	200	400	600	800	1000	Observed range in CPA
	predicted dbh (in.)								
40	4.4	5.4	6.5	7.9	9.6	—	—	—	(26–387)
50	5.2	6.3	7.6	9.3	11.2	12.6	—	—	(26–539)
60	—	7.2	8.7	10.5	12.8	14.3	15.5	—	(48–625)
70	—	—	9.7	11.7	14.2	15.9	17.2	18.4	(133–1146)
80	—	—	—	12.9	15.6	17.5	18.9	20.2	(216–1018)

where CR_{max} = the widest crown radius and CL = crown length (to the lowest living whorl). All parameters are highly significant ($P = 0.01$). Figure 3 shows that the CR at a given CL increases slightly with H, indicating that pines tend to develop slightly "flatter" crowns with increasing heights.

After solving Equation (9) to determine the *CPA* required to achieve a desired *D*, *CPA* can be converted to the equivalent *CR* and Equation (10) solved for *CL* (Appendix B) to predict how long crowns must be to achieve this radius. This relationship governs how much vacant growing space must be created by prior thinning or crop-tree release in order to prevent competition-caused death of lower branches above this point.

Stocking Guide

If the main goal is predicting individual-tree development, then Equation (8) or (9) should suffice. However, foresters commonly need to evaluate stocking or determine treatment response at the stand level. For this purpose, Equation (8) was translated from *D* and *CPA* to basal area (*BA*) and trees per acre (*TPA*), respectively, using the procedures first applied by Krajiceck et al. (1961) in their development of the CCF. These workers converted a crown width-dbh regression to a stand-level nomogram of basal area, trees per acre, and average diameter, by dividing the circular *CPA*s into 43,560 to determine the number of trees that would reach full crown closure at a given dbh. The same procedure has since been used by all stocking guides that use the *CCF* to determine the *B* level or lower limit of site occupancy, including the white pine stocking guide of Philbrook et al. (1973).

The height-dependent stocking curves (Figure 4) show the possible combinations of *BA* and *TPA* that will achieve full occupancy of the crown space at stand heights of 40 through 80 ft. In this case, stand height represents the average of all crop trees that reach this stage development. Be-

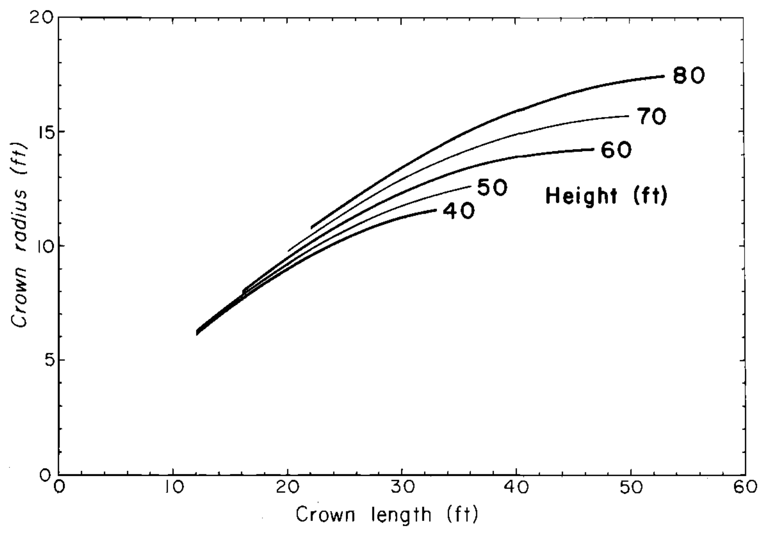

FIGURE 3. Relationship between crown radius, crown length, and total height for eastern white pine [from Equation (10)].

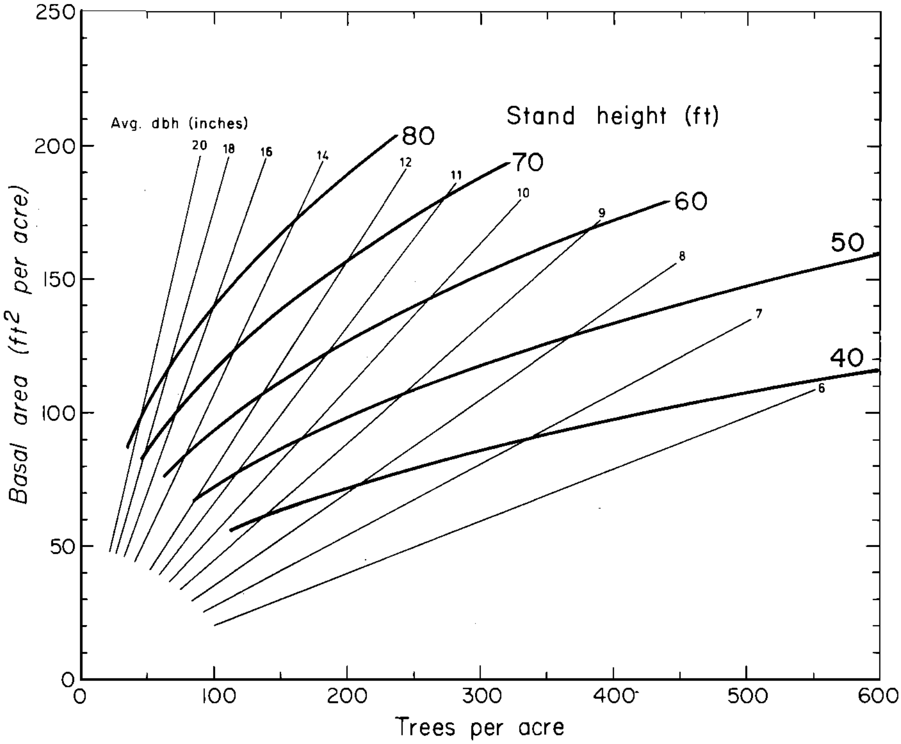

FIGURE 4. Height-dependent stocking guide for eastern white pine. Curves labelled "stand height" give possible combinations of basal area and trees per acre that will form a closed canopy at heights of 40 through 80 ft. [Calculated from Equation (9) assuming circular crown areas.]

cause most crop trees are dominants and codominants, this average height would normally be quite similar to that of site index trees. Curves for each height begin at the lowest observed *TPA* (or largest observed *CPA*) represented in the data for each 10-ft height class. This defines the smallest combined *BA-TPA* that completely occupies the crown space at a given height. This lower limit is thus identical in concept to the B level on stocking guides based on the CCF.

The stand-height curves are terminated arbitrarily at 80% of the maximum *BA* for each height given in Frothingham's (1914) normal yield table for site II. Philbrook et al. (1973) used a similar procedure to derive the upper limit (A level) on their pine stocking guide. Any combination of *BA* and *TPA* also determines average stand *D;* these are shown as straight lines radiating from the origin.

Discussion

If the B level of Philbrook et al. (1973) is plotted on the guide presented above, a large discrepancy is apparent in the position of the lower limit of complete crown closure (Figure 5). The height-dependent curves derived above suggest that crown closure can occur well below the B level of Philbrook et al. (1973), at densities that would be considered quite understocked by their criteria.

For example, Philbrook et al. (1973) B level requires about 90 trees to

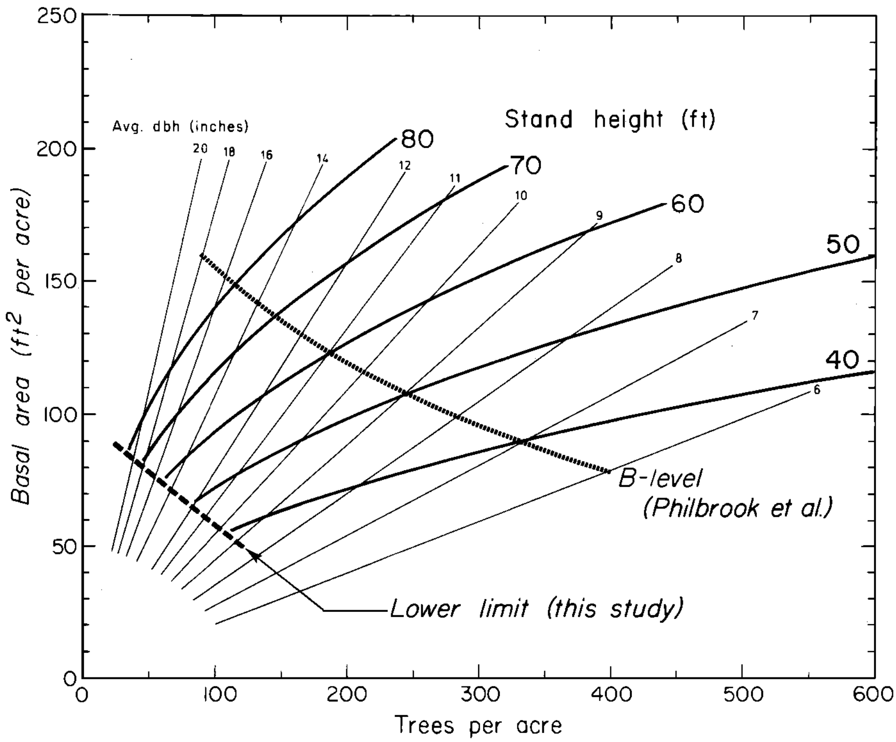

FIGURE 5. Comparison between B level from Philbrook et al. (1973) and lower limit of complete crown closure found in this study.

reach crown closure at an average dbh of 18 in. (Figure 5). According to Equation (9) this cannot be achieved until the trees are about 90 ft tall. However, Figure 5 [or Equation (9)] also shows that if thinnings are begun early enough to allow crop-tree crowns to cover 930 ft^2, one could grow 47 18-in. trees per acre at a height of only 70 ft. Similarly, the B level of Philbrook et al. requires 245 9-in. trees to achieve full stocking, which Equation (9) predicts would occur at a height of 50 ft. However, Figure 5 shows that, at the same height, full crown closure can also be achieved with as many as 596 7-in. trees or as few as 86 12-in. stems. In general, the CCF approach appears to overestimate the density required to achieve full crown closure in stands where trees develop large crowns (and thus, large diameters) early in stand development.

To analyse this apparent discrepancy between the two procedures for defining the minimum stocking level, it is helpful to review the rationale behind the B level as the lower limit of crown closure. The concept apparently originated with Gingrich (1964, 1967) who based the B level for upland oak on the CCF (Krajicek et al. 1961). CCF equations are derived by regressing crown width (*CW*) on *D*, squaring both sides, and multiplying by $\pi/4$ to convert linear tree dimensions to an area basis:

$$\pi/4\ CW^2 = CPA = (b_0 + b_1 D)^2 \qquad (11)$$

Since the CCF equation is derived from either open-grown or forest-grown dominant trees, it is supposed to define an absolute minimum full-stocking condition, under the assumption that crowns of such trees cover more area than those of the same dbh occupying less dominant positions.

Equation (11) shows that the CCF, used to derive B levels in all stocking guides for eastern species including that in Philbrook et al. (1973), assumes that height does not affect the relationship between *CW* and *D*. In contrast, our analysis [Equation (8) and Figure 2] and many other studies (Briegleb 1952, Bonner 1964a, 1964b, Smith and Bailey 1964), clearly demonstrate that height is a highly significant predictor. This difference is illustrated in Figure 6, which compares the height-dependent *CW-D* relationships found in this study with the B level in Philbrook et al. (1973). Evidently, Philbrook's (1971) equation (derived from 61 dominant, forest-grown pines in New Hampshire) would fit our data reasonably well if the relationship between *CW* and *H* were ignored. However, excluding height treats variation about this line [which is substantial in Philbrook's data (r^2 = 0.62)] as random experimental error, when, as we have shown [Equation (8)], much of this variation in *D* is explainable by including height in the regression. Figure 6 clearly shows that a tree can grow to a given *D* through many inversely related combinations of *CW* and *H*, presumably by producing either many small photosynthetic surfaces (tall, narrow-crowned trees) or fewer large ones (short, wide-crowned trees).

Because CCF equations such as Philbrook's *CW-D* relationship apply specifically to the *average* height for each dbh class, their utility for defining the *lower limit* of full stocking is questionable. For a given dbh, Philbrook's (1971) equation overpredicts the number of trees per acre at shorter-than-average heights, and underpredicts for taller trees. Consider the options for growing 12-in. dbh trees. Philbrook's equation applies specifically to a tree about 65 ft tall, with a crown area of 263 ft^2 (CW = 18.3 ft). Using this to define the B level indicates that fewer than 166 12-in. trees will not fully occupy the crown space. However, our data show that pines can reach 12

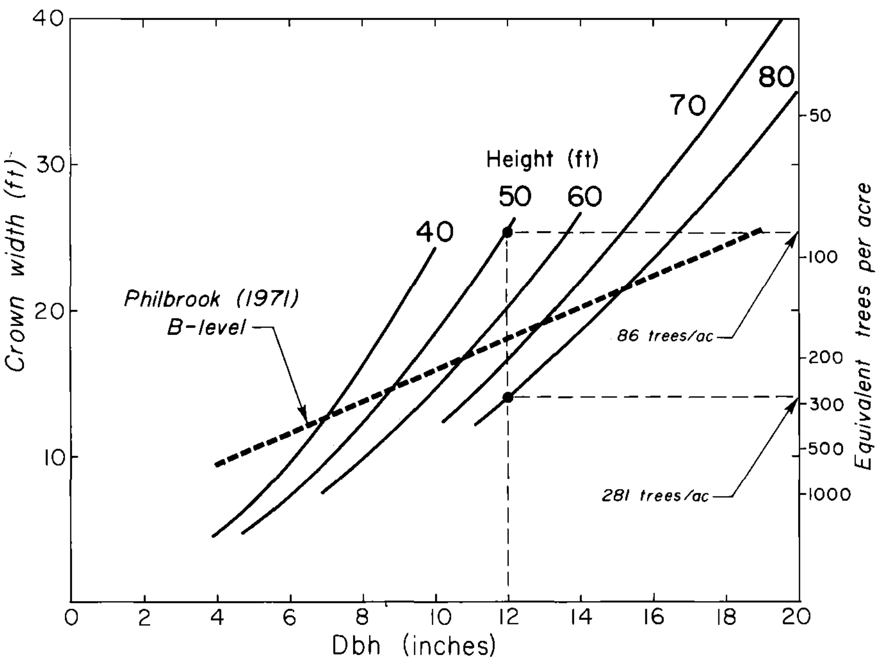

FIGURE 6. Comparison of the crown width-dbh regression used to determine the white pine B level (Philbrook 1971) with the height-dependent relationship used to derive Figure 4.

in. dbh at heights as short as 50 ft if crowns cover 506 ft^2 (CW = 25.3 ft). At this point only 86 trees per acre will fully occupy the crown space (Figure 6). The option also exists to grow as many as 281 12-in. pines per acre (CW = 14.1 ft), but one must wait until the trees are 80 ft tall.

The fact that the height-dependent stocking curves extend well below the B level (Figures 5 and 6) suggests that strict adherence to the B level would rule out many biologically feasible, low-density thinning schedules designed to achieve maximum diameter growth. To illustrate this point, consider two thinning schedules: one designed to grow 20-in. pines on a rotation that ends at a height of 80 ft; and another that reduces density to the B level at the same intervals. In both cases, assume that the average dbh is the same before and after thinning (i.e., $d/D = 1.0$).

To reach the 20-in. goal, solution of (9) shows that the required $CPA = 1265\ 20^{3.589}\ 80^{-2.514} = 971$ ft^2 or 45 trees per acre. To achieve the equivalent crown radius (17.6 ft), crown length would have to be 56.6 ft (from Appendix B). One would therefore have to thin when the live crown base has retreated to 23.4 ft (80 − 56.6) in order to prevent further death of branches above this point.

Assume the crown base is at 23.4 ft when stand height is 35 ft; crown length thus would be 35 − 23.4 = 11.6 ft (live-crown ratio = 33%). The corresponding crown radius [from Equation (10)] would be 5.93 ft, or a CPA = 110 ft^2 (390 trees per acre). Solution of Equation (8) shows that dbh would average 6.1 in. at this stage, very close to the B level of Philbrook et al. (1973).

Residual density at this initial thinning depends on how many subsequent thinnings are planned. Assume two others at 15-ft height intervals, to occur at heights of 50 and 65 ft. The next step is to calculate CL at the time of the next thinning, and determine the corresponding CR from Equation (10). If the crown base remains at 23.4 ft, CL would be 50 − 23.4 = 26.6 ft. This corresponds to a CR = 11.1, or 113 trees per acre at full crown closure (CPA = 385). One must therefore thin to this residual density to prevent further retreat of the live crown base. The expected D can now be determined by substituting 385 for CPA and 50 for H into (8), which gives D = 11.1 in.

When stand height reaches 50 ft, crown closure should again occur, and another thinning will be necessary. Repeating the process above for H = 65 (the time of the next thinning) shows that CL will be 41.6, CR = 14.6 and CPA = 670. The required residual density is 65 trees per acre, which will be 15.6 in. dbh when they reach a height of 65 ft. The final thinning then is made to a residual density of 45 trees per acre, which should produce a D of 20.0 in. when crowns again close (CR = 17.6, CPA = 971) at a height of 80 ft.

Now, consider a conventional thinning schedule where thinnings to the B level are made at the same intervals as the low-density schedule above. Since the initial stand density is already at the B level, no thinning is made until H = 50. As trees grow from 35 to 50 ft in height, crown radius cannot expand, so CPA (= 111) does not change as the crown base retreats upward. At H = 50 when the first thinning is made to the B level of 290 trees per acre, solution of (8) shows that D = 7.9 in. When the stand height reaches 65 and crown closure has again occurred (at a CPA = 150), D = 10.3. After a second thinning to the corresponding B level of 205 trees per acre, trees again achieve full crown closure at H = 80, at which point the predicted D = 13.1 in.

When both thinning schedules are diagrammed on the stocking guide (Figure 7), the low-density schedule appears to drastically understock the stand relative to the conventional pine B level. Yet, trees achieve full crown closure at the desired heights and reach the dbh goal. As expected, the conventional B-level schedule produces more total volume (Table 2), but the average *D* at the end of a 60-yr rotation is nearly 7 in. smaller. Determining which scenario is better requires a detailed financial analysis, including the effect of tree size and quality on clear-lumber yields and logging costs. The important point is that *both* schedules are biologically feasible, yet only one would be considered using the conventional B-level approach. Indeed, the goal of 20-in. trees in 60 years cannot be met by any strategy which allows the crown base to recede above 23.4 ft.

Our purpose in redefining the lower limit of crown closure is strictly to establish the densities at which *individual-tree* diameter growth should theoretically begin to decline due to restricted crown development. We do not intend to imply, as Mar:Moller's (1947) hypothesis would suggest, that total stand basal-area or cubic volume growth remains constant over a wide range in density between the critical lower threshold and the upper limit. Leak (1981) demonstrated that growth of red and white pine stands continued to increase above the B level, with maximum growth near the A-level. Baskerville (1965) found that total biomass production of balsam fir increased linearly from the minimum to maximum naturally occurring densities. Thus, one should expect to sacrifice total volume production under a thinning schedule that maintains densities near the lower ends of the height-dependent stocking curves in Figure 4, as was evident in the comparison in Table 2. This does not obviate accurate definition of the lower limit of crown closure, however. This point is critical to prevent a stand from being thinned too heavily and truly understocked in the sense that it can never achieve full crown closure at the residual density.

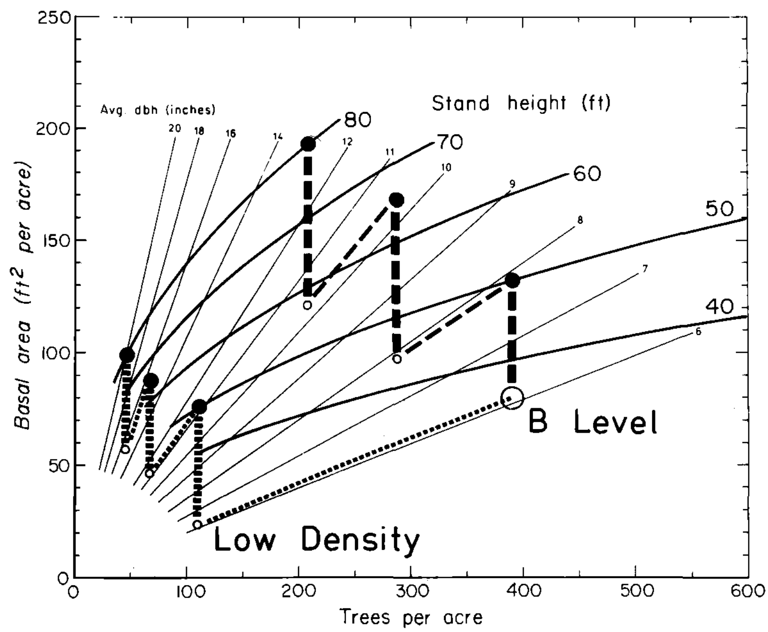

FIGURE 7. Stand growth resulting from low-density and B level thinning schedules (described in text), diagrammed on stocking guide.

TABLE 2. Predicted tree and stand parameters resulting from the low-density (LD) and B Level (BLV) thinning schedules described in text.

	Height (ft)						
	35	50		65		80	
	Both	LD	BLV	LD	BLV	LD	BLV
dbh (in.)	6.1	11.1	7.9	15.6	10.3	20.0	13.1
Crown radius (ft)	5.9	11.1	5.9	14.6	6.9	17.5	8.2
Crown length (ft)	11.6	26.6	11.3	41.6	13.2	56.6	15.8
Trees/ac							
Initial	390	113	390	65	290	45	205
(Residual)	(113)[a]	(65)	(290)	(45)	(205)	—	—
Basal area (ft²/ac)							
Initial	79	76	133	86	168	98	192
(Residual)	(23)[a]	(44)	(99)	(60)	(119)	—	—
Stand volume[b]							
Cubic ft/ac (3.0-in.-top inside bark)							
Initial	1788	1676	3283	2288	4750	3419	6312
(Residual)	(518)[a]	(964)	(2441)	(1584)	(3358)	—	—
Board ft/ac (6.0-in.-top inside bark)							
Initial	0	6758	0	13505	20612	20505	36030
(Residual)	0	(3887)	0	(9350)	(14570)	—	—

[a] LD schedule only; no thinning is made in BLV until height = 50.
[b] Calculated from equations in Leak et al. (1970), assuming Girard Form Class = 80.

Because the CCF and all published stocking guides apparently used the assumption of circular crowns to establish the B levels, we retained this procedure strictly to facilitate comparison with our height-dependent approach. Obviously, it is geometrically impossible for circular crowns to fully occupy the crown space without overlap or gaps; the main virtue of this simple model of crown competition is computational expediency. Using 43,560 ft^2 of circular crown area per acre does not assume complete crown closure, but rather a situation where crowns overlap by an amount that equals the area in interstices not occupied by branches. Under competition, tree crowns will touch well before all intersticial space is occupied. It can be shown that the area occupied in this situation, assuming perfect triangular spacing, equals 90.7% of the stand area; the equivalent value for square spacing is only 78.5%. To achieve full crown cover, branches must continue to develop into the interstices, during which other branches will either intertwine (creating overlapping *CPA*s) or cease to grow (creating polygon-shaped *CPA*s). Leak (1983) suggests that the assumption of square crowns more accurately reflects closed-stand conditions, and recalculated new B levels for several northern hardwood species on this basis. The question then arises whether field procedures used to determine crown width have measured the diagonals of the square, or the central axes parallel with the sides.

While they provide a useful point of departure for quantifying stocking, guides that use the CCF to define the critical lower limit of site occupancy may warrant reexamination. Excluding height from the crown width-dbh relationship and assuming that crown width alone controls dbh, oversimplies the true three-dimensional relationship between crown size and stem growth. Also, incorporating height into the crown width-dbh regression allows the forester to select target tree sizes at specific points in the rotation defined by tree height and crown projection area. With an appropriate equation such as (10), crown length required to achieve the desired CPA at a given height then can be determined, and specific thinning schedules designed accordingly. This is a major advantage over the basic CCF equation, which cannot be evaluated relative to height or age without supplementary information.

In practice, guides based on the CCF can lead to thinning strategies that are far too conservative if the objective is maximum diameter growth of carefully selected, high-value crop trees. For species such as white pine, which command a high premium for large high-quality sawlogs, strict adherence to a CCF-based B level could exclude lucrative, low-density thinning schedules that fully occupy the crown space if thinnings are begun early enough to develop wide crowns.

We encourage researchers with appropriate data to refit the CCF regressions for species other than white pine to determine if they too are height-dependent. Results could readily be incorporated into existing guides, retaining the familiar format. It is important that data used to derive CCF equations include the widest possible range in crown widths at each height. These conditions might have to be created by seemingly drastic crop-tree release thinnings at an early age. CCF equations derived solely from dense, natural stands probably do not represent the full range of possible tree sizes that can result from low-density thinning regimes.

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Appendix A. Derivation of stocking equation.

Substituting (5) and (6) into (1) gives:

$$a\,D^bH^c = d(CPA\ H)^{k_1}, \text{ where } d = k_0k_2{}^{k_1}$$

Isolating D on the left-hand side:

$$D^b = d(CPA\ H)^{k_1}/(a\ H^c)$$

or

$$D^b = (d/a)\ CPA^{k_1}H^{(k_1-c)}.$$

Raising both sides to the power $(1/b)$ gives:

$$D = (d/a)^{(1/b)}CPA^{(k_1/b)}H^{(k_1-c)/b}$$

or

$$D = b_0CPA^{b_1}H^{b_2} \text{ where } b_0 = (d/a)^{(1/b)},\ b_1 = k_1/b \text{ and } b_2 = (k_1 - c)/b$$

Appendix B. Solution of (10) for crown length (*CL*) in terms of crown radius (*CR*) and height (*H*).

$$CL = [-b + (b^2 + 4aCR)^{0.5}]/2a$$

where

$$a = 0.00006024\ H - 0.009988$$

$$b = 0.6027$$